## Multiple Reactions

We will define several terms that are used in material balance calculations when chemical reactions are involved.
$100 \mathrm{~mol} / \mathrm{s}$ of equimolar mixture containing ethylene and oxygen is fed to a reactor where they combine to make ethylene oxide. The fractional conversion of ethylene is $20 \%$. Find flow rates of all components in reactor exit stream at steady state.

First write balanced chemical equation

$$
2 \mathrm{C}_{2} \mathrm{H}_{4}+\mathrm{O}_{2} \longrightarrow \mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}
$$

Then, draw a flowchart and label inlet and exit streams of the reactor.


If ethylene and oxygen are supplied to the reactor in the mole ratio of 2 , we say that they are fed in stoichiometric ratio (see the equation above) and they are in stoichiometric quantities. The reactant present in less than stoichiometric quantity is called limiting reactant. In the above example, oxygen is the limiting reactant because its stoichiometric quantity is 100 moles of oxygen.

Fractional conversion, $\mathrm{f}=\frac{\text { Moles reacted }}{\text { Moles fed }}$

We define extent of reaction as number of moles reacted and use the symbol $\xi$ for it.

Write balance of $\mathrm{C}_{2} \mathrm{H}_{4}$ :

$$
\text { in }- \text { out }+ \text { generation }- \text { consumption }=\text { accunulation }
$$

Why have generation and accumulation terms been canceled?

$$
50(\mathrm{~mol} / \mathrm{s})-\mathrm{n}_{2}(\mathrm{~mol} / \mathrm{s})-\xi(\mathrm{mol} / \mathrm{s})=0
$$

$\xi$ is number of moles of ethylene reacted. In this problem,

$$
\xi=0.2 \times 50 \mathrm{~mol} / \mathrm{s}=10 \mathrm{~mol} / \mathrm{s}
$$

Therefore, $\mathrm{n}_{2}=40 \mathrm{~mol} / \mathrm{s}$.
Balance of oxygen: $\quad 50 \mathrm{~mol} / \mathrm{s}-\mathrm{n}_{3} \mathrm{~mol} / \mathrm{s}-\xi / 2 \mathrm{~mol} / \mathrm{s}=0 \quad$ or $n_{3}=45 \mathrm{~mol} / \mathrm{s}$.
You could write balance of $\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}$ also.
Now, in the same reactor an additional reaction in which $\mathrm{C}_{2} \mathrm{H}_{4}$ could combine with $\mathrm{O}_{2}$ to produce $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ can also occur. This reaction is side reaction and the products of this reaction are undesired products. $\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}$ is the desired product.

$$
\mathrm{C}_{2} \mathrm{H}_{4}+3 \mathrm{O}_{2} \longrightarrow \mathrm{CO}_{2}+\mathrm{H}_{2} \mathrm{O} \quad \text { side reaction }
$$

If fractional conversion remains same at $20 \%$, we want to find molar flow rates of components in exit stream. Draw a flowchart and label the streams.

$\xi_{1}=$ extent of reaction in reaction producing $\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}$
$\xi_{2}=$ extent of reaction in reaction producing $\mathrm{CO}_{2}$

$$
\mathrm{f}=\frac{\xi_{1}+\xi_{2}}{50}=0.2 \text { or } \xi_{1}+\xi_{2}=10 \mathrm{~mol} / \mathrm{s}
$$

You could write balances of all species involved and organize it in the following way

| Species | balance equation |
| :--- | :--- |
|  |  |
| $\mathrm{C}_{2} \mathrm{H}_{4}$ | $50-\mathrm{n}_{2}-\xi_{1}-\xi_{2}=0$ |
| $\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}$ | $-\mathrm{n}_{1}+\xi_{1}=0$ |
| $\mathrm{O}_{2}$ | $50-\mathrm{n}_{3}-\xi_{1} / 2-3 \xi_{2}=0$ |
| $\mathrm{CO}_{2}$ | $-\mathrm{n}_{4}+2 \xi_{2}=0$ |
| $\mathrm{H}_{2} \mathrm{O}$ | $-\mathrm{n}_{5}+2 \xi_{2}=0$ |

$\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}$ is the desired product and $\mathrm{CO}_{2}$ is undesired. Obviously, we want to produce more of the former than the latter. To quantify, the relative production rates of desired and undesired products, we define

Moles of desired product
Selectivity $=\frac{\text { Moles of undesired product }}{\text { Mon }}$

Suppose selectivity $=4.5$ in the above problem, then $\frac{\xi_{1}}{2 \xi_{2}}=4.5$ or $\xi_{1}=9 \xi_{2}$
Combine fractional conversion and selectivity to find $\xi_{1}$ and $\xi_{2}$ and then the molar flow rates of all species.

Yield is another term that signifies the amount of desired product made:

$$
\begin{aligned}
\text { Yield }= & \begin{array}{l}
\text { Moles of desired product made if the limiting reacting reacts } \\
\text { completely }
\end{array}
\end{aligned}
$$

Yield of $\mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}$ in the above problem $=\xi_{1} / 50=0.18$ (check! why is 50 used in the denominator?)

What we did so far is balances of molecular species like $\mathrm{C}_{2} \mathrm{H}_{4}, \mathrm{C}_{2} \mathrm{H}_{4} \mathrm{O}, \mathrm{CO}_{2}, \mathrm{H}_{2} \mathrm{O}$ and $\mathrm{O}_{2}$.
Atomic species balance, that is balance of atomic C , atomic H and atomic O , could also be done.

For atomic species, input = output because atoms can neither be generated nor destroyed.
For the above problem,
Balance of atomic C: $50 \mathrm{~mol} / \mathrm{s} \mathrm{C}_{2} \mathrm{H}_{4} \times 2 \mathrm{~mol} \mathrm{C} / 1 \mathrm{~mol} \mathrm{C} 2 \mathrm{H}_{4}=2 \mathrm{n}_{1}+2 \mathrm{n}_{2}+\mathrm{n}_{4}$
Balance of atomic H: $50 \mathrm{~mol} / \mathrm{s} \times 4 \mathrm{~mol} \mathrm{H} / 1 \mathrm{~mol} \mathrm{C}_{2} \mathrm{H}_{4}=4 \mathrm{n}_{1}+4 \mathrm{n}_{2}+2 \mathrm{n}_{5}$
Balance of atomic O: $50 \mathrm{~mol} / \mathrm{s} \times 2 \mathrm{~mol} \mathrm{O} / 1 \mathrm{~mol} \mathrm{O}_{2}=\mathrm{n}_{1}+2 \mathrm{n}_{3}+2 \mathrm{n}_{4}+\mathrm{n}_{5}$
Two more equations, one for selectivity and one for fractional conversion, could be written and all unknowns could be solved for.

You will find that making atomic balances simplifies the analysis of material balance calculations (recall the problem on purge solved in the tutorials)

## Purge

If an inert material (which does not react) is present in a mixture of materials being fed to a reactor and the unreacted reactants are separated and recycled to the reactor, a part of the recycle stream is purged to prevent accumulation of the inert material in the process. Consider the following flowchart of a process with recycle and purge


Methanol is made by reacting carbon dioxide and hydrogen. The fresh feed to the process contains 0.67 mol of $\mathrm{N}_{2}$, which is an inert material because it does not react. Hence, it should be purged or bled from the recycle stream. If feed to the process does not contain $\mathrm{N}_{2}$, there is no need for purge.

## Conversions

Single pass conversion $=\frac{$\begin{tabular}{l}
Reactant input <br>
to the reactor

$-$

Reactant output <br>
to the reactor

}{

Reactant input <br>
to the reactor
\end{tabular}}

Reactant input _ Reactant output to the process - to the process
Overall conversion $=$

> Reactant input
> to the process

For the above flowchart,
single pass conversion of $\mathrm{CO}=\frac{29.00 \mathrm{~mol} \mathrm{CO}-24.95 \mathrm{~mol} \mathrm{CO}}{29.00 \mathrm{~mol} \mathrm{CO}}=14 \%$

Overall conversion of CO $=\frac{10.67 \mathrm{~mol} \mathrm{CO}-1.28 \mathrm{~mol} \mathrm{CO}}{10.67 \mathrm{~mol} \mathrm{CO}}=88 \%$

## Combustion reactions

These reactions involve burning fuel (solid, liquid and gas) with oxygen (or specifically air). Heat is generated because the reactions are exothermic. This heat is utilized in producing steam, which in turn drives turbines to generate electricity. Many power intensive chemical industries such as cement have captive power plants to ensure continuous supply of power.
Liquid fuels generally used in industry are heavy petroleum fractions and gaseous fuels are natural gas (mostly methane), hydrogen, ethane, propane, butane etc.

When carbon in fuels is converted to only carbon dioxide, we call the process as complete combustion; if CO is also produced, the process is partial combustion. Hydrogen is converted to water and sulphur is converted to sulphur dioxide.

Air (or oxygen) is supplied in more than stoichiometric quantities to make sure that all of the fuel fed to a combustion chamber reacts. In other words, the fuel is the limiting reactant. The following definitions are useful in combustion reactions:

Theoretical oxygen: amount of oxygen required for $100 \%$ conversion of fuel to carbon dioxide.
Theoretical air: amount of air corresponding to theoretical oxygen.
Moles air fed _ Theoretical moles of air
Excess air =
Moles air fed

Consider the following combustion process in which methane is burned. Find excess air.


$$
\mathrm{CH}_{4}+2 \mathrm{O}_{2} \longrightarrow \mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}
$$

Theoretical oxygen $=150$ moles even though only 140 moles of $\mathrm{O}_{2}$ are consumed for 70 mol of $\mathrm{CH}_{4}$ reacted. Keep this is mind whenever you make excess and theoretical air calculations.
Oxygen supplied $=180.6$ moles
Excess oxygen $=\frac{180.6-150}{180.6}=16.95 \%$

In the above process, even if some of the methane burns to produce CO, excess air and theoretical air will remain same. Excess and theoretical air or oxygen are always based on $100 \%$ conversion of fuel to $\mathrm{CO}_{2}$.

Sometimes, the composition of product gases of combustion is reported on a dry basis. This means that the composition does not take into account the amount of water present in the gas. Wet basis includes water. A gas containing $40 \% \mathrm{CH}_{4}$ and $40 \% \mathrm{H}_{2}$ and $20 \%$ $\mathrm{H}_{2} \mathrm{O}$ on a wet basis would contain $50 \% \mathrm{CH}_{4}$ and $50 \% \mathrm{H}_{2}$ on a dry basis.

## Test Your Understanding

1. Explain in your own words the meaning of purge
2. Check atomic species balance over the reactor and for the entire process in the problem described in this handout where ethylene combines with oxygen to produce ethylene oxide
3. In the problem on excess air, what is excess air if $2 \%$ of methane burned forms CO ?
4. In the problem on excess air, what is excess air if conversion of methane is $90 \%$ ?
5. What are single pass and overall conversions?
6. In the problem described in Purge section, what are single and overall conversions, if 5 moles of methanol are formed and 2 moles of CO are purged?
7. A gas contains 1 mole of $\mathrm{H}_{2}, 1 \mathrm{~mol}$ of $\mathrm{O}_{2}, 3 \mathrm{~mol}$ of $\mathrm{C}_{2} \mathrm{H}_{4}, 6 \mathrm{~mol} \mathrm{CO}_{2}$ and 6 mol of $\mathrm{H}_{2} \mathrm{O}$. What is the molar composition of this gas on a wet basis? On a dry basis?
