Units and Dimensions

Basic properties such as length, mass, time and temperature that can be measured are called dimensions. Any quantity that can be measured has a value and a unit associated with it. For example, density of 1000 kg/m^3 has units of kg/m^3 and a value of 1000.

Values of quantities can be added only if their units are same. You can do 2kg + 3 kg but not 2 kg + 3 g. In the latter, either kg should be converted to g or vice versa so that addition can be performed. Multiplication and division can be done on quantities with different units. Velocity multiplied by mass is momentum, 10 kg x 10 m/s = a momentum of 100 kg m/s.

Most often in engineering problems, especially in chemical engineering problems, units need to be converted from one system (SI/CGS/American) to another (SI/CGS/American).

To convert to new unit from old unit, simply multiply by old unit with a conversion factor, which is new unit/old unit.

Density = 0.8 g/cc. What is the density in kg/m^3?

$$\text{Density} = \frac{0.8\ \text{g}}{\text{cc}} \left| \frac{1\ \text{kg}}{1000\ \text{g}} \right| \frac{10^6\text{cc}}{\text{m}^3} = 800\ \text{kg/m}^3$$

(note that old units cancel out)

Specific heat of water is 1 cal/ (g ^{o}C). Let's convert the units to J/ (kg ^{o}C)

$$\text{Cp} = \frac{1\ \text{cal}}{\text{g}\ ^o\text{C}} \left| \frac{4.184\ \text{J}}{1\ \text{cal}} \right| \frac{1000\ \text{g}}{1\ \text{kg}} = 4184\ \text{J/kg}\ ^o\text{C}$$

Convert acceleration of 9.8066 m/s^2 into units of ft/s^2. You need to know certain basic conversions such as meters to feet, centimeter, millimeter; Joules to calorie.

1 N is the force required to accelerate a mass of 1 kg at 1m/s^2. In American system of units, a unit of force is pound force (lb_f). It is the force required to accelerate a mass of 1 pound (lb_m) at 32.174 ft/s^2, which is the acceleration due to gravity at sea level and 45^o latitude.

$$1\ \text{N} = 1\ \text{kg m/s}^2$$
$$1\ \text{lb}_f = 32.174\ \text{lb}_m\ \text{ft/s}^2$$

1 kg_f (kilogram force) is the force required to accelerate 1 kg mass at 9.81 m/s^2. That is 1 kg_f = 9.81 N.

1 atmosphere pressure is 1.013×10^5 N/m^2. What is the value when the units are lb_f/in^2?

1.013×10^5 N/m^2 = 1.013×10^5 kg m/ (s^2 m^2) (Recall: 1 N = 1 kg m/s^2)

$$1.013 \times 10^5 \; \frac{\text{kg m}}{\text{s}^2} \left| \frac{2.2\ \text{lb}_m}{1\ \text{kg}} \right| \frac{3.281\text{ft}}{1\ \text{m}} \left| \frac{1}{1\ \text{m}^2} \right| \frac{1\ \text{m}^2}{39.37\ \text{in}^2} \left| \frac{1\ \text{lb}_f}{32.174\ \frac{\text{lb}_m\ \text{ft}}{\text{s}^2}} \right.$$

$$= 14.7 \ \frac{\text{lb}_f}{\text{in}^2}$$ or psi (pounds per square inch)

Dimensional Homogeneity

Additive terms in all valid equations should have same dimensions.

In the equation, $s = ut + \frac{1}{2}at^2$, all terms have dimensions of m (check). Dimensional homogeneity helps in finding dimensions of some terms in an equation if dimensions of some other terms are known. For example, consider van der Waals equation of state for a real gas:

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

Inspection reveals that the second term in the first bracket should have units of pressure. If pressure is in atm and volume is in m^3/mol, a must have units of atm m^6/mol^2. Similarly, units of b are m^3/mol.

Note that arguments of exponential, logarithmic, trigonometric functions are dimensionless.

Notation, Significant figures and precision (See Felder's and Rousseau's book)

Significant figures of a number are the digits from the first non-zero digit on the left to either a) the last digit on the right if there is a decimal point or b) the last non-zero digit if there is no decimal point. Few examples are considered below:

1239 has four significant figures

1.00×10^2 has three significant figures

1.0×10^2 has two significant figures

0.059 or 5.9×10^{-2} has two significant figures

0.05900 or 5.900×10^{-2} has four significant figures

The number of significant figures indicates the precision with which a variable can be measured; the more the number of significant figures the higher is the precision.

Certain rules need to be followed when adding, subtracting, multiplying or dividing two numbers. When two numbers are added or subtracted, note the position of the last significant figure of each number relative to the decimal point. The one farthest to the left is the position of the significant figure of the sum or the difference. For example,

$$\begin{array}{rr} & 1290 \\ + & 2.45 \\ \hline & 1292.45 \\ \hline \end{array}$$

arrows indicate the last significant figure

this has to be rounded off to 1290

$1.000 + 0.036 + 0.22 = 1.2560$ this has to be rounded off to 1.26

When numbers are multiplied or when one is divided by the other, the last significant figure of the result is the lowest of the last significant figure of the multiplicands or divisors. Consider the following example,

$\overset{3}{(3.57)}\overset{4}{(4.286)} = \overset{7}{15.30102}$, should be rounded off to three significant figures to 15.3 (numbers above the multiplicands indicate number of significant figures).

$\overset{2}{(5.2 \times 10^{-4})}\overset{4}{(0.1635 \times 10^{7})}/\ \overset{3}{(2.67)} = \overset{9}{318.426966} = 3.18426966 \times 10^2$, must be rounded off to two significant figures to $3.2 \times 10^2 = 320$.

Test Your Understanding

1. How many kg_f/cm^2 is one atmospheric pressure?
2. 14.7 psi = ——— kg_f/cm^2.
3. The dependence of second order rate constant of a reaction is given by

$$k = k_o \exp\left(-\frac{E}{RT}\right)$$

   what are the units of k_o and that of E if R is in cal/ (mol K) and T is in Kelvin, K.

4. Find the result in the following
   a) 1.000 + 10.2 b) 18.76 – 7 c) $(1.76 \times 10^4)(0.12 \times 10^{-6})$
   d) (5.74)(38.27)/(0.001250)

5. Find relation between Newton and lb_f.

Density and Specific gravity

Density is mass per unit volume and specific volume is inverse of density; it is volume occupied by unit mass. Specific gravity is the ratio of density of a given substance to the density of a standard substance. For liquids, the standard substance is water at 4 degrees where its density is 1 g/cc or 1000 kg/m^3.

Flow rate

It is the mass or volume or moles that pass cross section in unit time. Units of mass flow rate are kg/s, g/s so on, units of volume flow rate are m^3/s, ft^3/s, cc/s, liters/s so on and units of molar flow rate are g mol/s, lb mol/s, k mol/s and so on. You should be able to convert units from one set to another by now.

Mass, molar and volume flow rates are not independent and they are related

$$m = \rho V$$

m is mass flow rate, ρ is density and V is volume flow rate. Molar flow rate is mass flow rate divided by molecular weight.

Chemical Composition

Molecular weight of a substance is the sum of atomic weights of atoms that constitute it. A gram mole written as g mol is the amount of species in grams equal to its molecular weight. A pound mole written as lb mol is the amount of species in pounds (lbs) equal to its molecular weight. A kilo mole written as k mol is the amount of species in kilograms equal to its molecular weight. A ton mole is the amount of species in tons equal to its molecular weight.

The conversion between molar units is same as the conversion between mass units. That is

1 lb = 453.6 g	1 kg = 1000 g	1 kg = 2.2 lbs
1 lb mol = 453.6 g mol	1 kg mol = 1000 g mol	1 kg mol = 2.2 lb mol

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You should be able to express composition in mass fractions of a mixture if the composition is given in mole fractions or vice versa.

Composition of air is 79 mole% nitrogen and 21 mole% oxygen. Find the composition in mass percentages.

To solve problems of this kind, assume a basis of 100 moles or 100 g or 100 kg. In the above problem, choose 100 moles as basis because mole fractions are specified.

Mass fraction of nitrogen = $\dfrac{79gmol\dfrac{28gN_2}{gmol}}{79gmol\dfrac{28gN_2}{gmol}+21gmol\dfrac{32gO_2}{gmol}}=0.767$

Mass fraction of oxygen = 1 – mass fraction of nitrogen. (Sum of mass fractions of all species add up to 1) = 1 – 0.767 = 0.232

Concentration of liquids is usually expressed as molarity (moles/liter) or normality (equivalents/liter).

Concentration of gases, usually pollutants, which are present in small quantities are expressed in ppm (parts per million) or ppb (parts per billion). Million is 10^6 and billion is 10^9. ppm is the number of parts of a species in million parts of the mixture.

$$\text{ppm (parts per million)} = \frac{\text{parts of species}}{\text{million parts of mixture}}$$

ppb is the number of parts of a species in billion parts of the mixture.

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For gases, parts usually mean moles. For liquids, it is generally mass (g or kg or lbs).

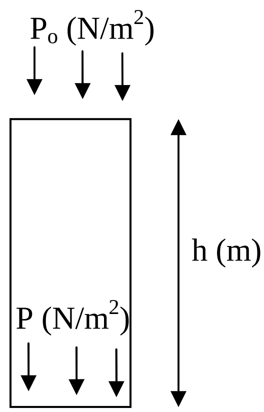
Relationship between mole fraction, ppm and ppb.

ppm = mole fraction x 10^6; ppb = mole fraction x 10^9

Pressure

Pressure is force per unit area. Units of pressure are N/m^2, $dynes/cm^2$, lbf/in^2. The SI unit is Pascal (Pa), which is N/m^2.

The pressure exerted by a liquid contained at the base of a column is called hydrostatic pressure.



$P = P_o + \rho gh$. Pressure is sometimes expressed as the head of a fluid in units of length (m, ft, cm etc). The relation between pressure and head can be obtained by thinking of an imaginary column filled with fluid that would exert the given pressure at the base if the pressure at the top, P_o, is set to zero.

$P = \rho\ g$ (head). If P is set to atmospheric pressure = 1.013×10^5 Pa, head of water = 11.24 m. That is, water filled up to 11.24 m exerts atmospheric pressure at the base of an imaginary column.

Sometimes, pressure is given in terms of gauge pressure. It is absolute pressure less atmospheric pressure.

$$P_{gauge} = P_{absolute} - P_{atmospheric}$$

Gauge pressure is what you read on instruments when you fill air in vehicles in petrol stations. Gauge pressure can be negative when absolute pressure is less than atmospheric pressure. Absolute pressure of 75 cm of mercury corresponds to -1 cm of gauge pressure. It is also referred to as 1 cm of vacuum.

Temperature

The common temperature scales are:

Celsius scale: freezing point of water is 0^oC and boiling point is 100^oC at a pressure of 1 atm. Lowest attainable temperature is -273.15^oC.

Fahrenheit scale: freezing point of water is 32^oF and boiling point is 212^oF at a pressure of 1 atm. Lowest attainable temperature is -459.67^oC.

Kelvin scale is chosen to make lowest temperature in Celsius scale equal to 0 and Rankine scale is chosen to make lowest temperature in Fahrenheit scale equal to 0.

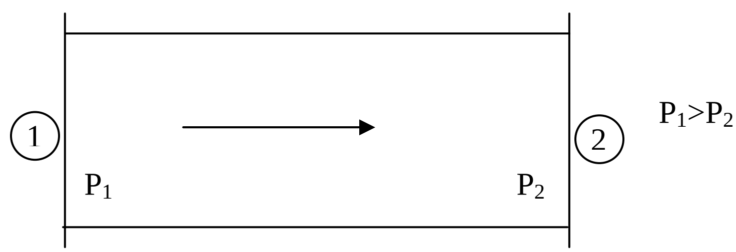
Thus, T (K) = T (^oC) + 273.15 and T (^oR) = T (^oF) + 459.67.

Also, T (^oF) = 1.8 x T (^oC) + 32

TEST YOUR UNDERSTANDING

1. What is the volumetric flow rate of water if the mass flow is 10 g/s? What is its molar flow rate?

2. A gas flows through a pipe and its pressure drops from P_1 to P_2. Compare the volumetric flow rates at points 1 and 2. (Remember that mass is conserved). What is the comparison if liquid is flowing and its density remains constant?



3. Find the relation between T (^oR) and T (K); Which is warmer: a rise of 5^oC or 5^oF?
4. Express atmospheric pressure in inches of mercury.
5. Compute pressure in atm in ideal gas equation if vacuum is a) 50 cm Hg b) 0.5 kgf/cm^2
6. Specific heat of carbon dioxide is a polynomial function of temperature. The function is:

$$Cp = 8.448 + 0.5757 \times 10^{-2}\ T - 0.2159 \times 10^{-5} T^2 + 0.3059 \times 10^{-9} T^3$$

Where Cp is in Btu/ (lb mol ^oF). Find the function if Cp is expressed in J/ (gmol K).

1 Btu = 252 cal.