Indian Institute of Technology, Bombay
Department of Chemical Engineering
CL-692, Digital Control

Test 2 **Maximum Marks: 100**
19 October 2006 **17:05-18:30 Hrs**

Note:
No borrowing of anything is allowed.

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1. For PID controllers, we derived the condition for offset free tracking of step changes as,

$$S_c = T_c$$

In the 2-DOF pole placement controller, we have to use γT_c in the place of T_c. The above condition becomes,

$$S_c(1) = \gamma T_c(1)$$

Because $S_c = A^g S_1$ and $T_c = A^g T_1$, it becomes,

$$S_1(1) = \gamma T_1(1)$$

But, $T_1 = 1$. In view of this, the required condition is,

$$S_1(1) = \gamma$$

2. (a) 2-DOF pole placement controller:

$$\begin{aligned}
x + jy &= -0.5 \pm j0.5 \\
\rho e^{j\omega} &= \frac{1}{\sqrt{2}} e^{j3\pi/4} \\
\omega &= 3\pi/4 \\
\rho &= 1/\sqrt{2} \\
\phi_{cl} &= (1 - \rho e^{j\omega})(1 - \rho e^{-j\omega}) \\
&= 1 - z^{-1} 2\rho \cos\omega + z^{-2}\rho^2 = 1 + z^{-1} + 0.5z^{-2} \\
A^b &= B^b = B^g = k = 1
\end{aligned}$$

Aryabhatta's identity:

$$R_1(1 - z^{-1}) + z^{-1} S_1 = 1 + z^{-1} + 0.5z^{-2}$$

Solution is,

$$\begin{aligned}
R_1 &= 1 - 0.5z^{-1} \\
S_1 &= 2.5 \\
R_c &= R_1\Delta = (1 - 0.5z^{-1})(1 - z^{-1}) \\
S_c &= S_1 A^g = 2.5(1 - 0.9z^{-1}) \\
T_c &= T_1 A^g = 1 - 0.9z^{-1} \\
\gamma &= S_1(1) = 2.5
\end{aligned}$$

Control law:

$$u = \frac{S_c}{R_c} e = \frac{2.5(1 - 0.9z^{-1})}{(1 - 0.5z^{-1})(1 - z^{-1})}$$

(b) Control law in Sec. 8.3.5 can be used. We obtain,

$$\begin{aligned}
r_1 &= -0.5 \\
s_0 &= 2.5 \\
s_1 &= -2.25 \\
s_2 &= 0
\end{aligned}$$

The only solution is $N = -1$. Take $T_s = 0.1$. To determine K, τ_i, τ_d:

$$r_1 = \frac{\tau_d}{\tau_d - 0.1} = -0.5$$

Solving,

$$\tau_d = 0.1/3$$

From other two values, we obtain,

$$\begin{aligned}
s_0 &= K(1.5 + \frac{0.1}{\tau_i}) = 2.5 \\
s_1 &= K(-0.5 - \frac{0.1}{2\tau_i}) = -2.25 \\
s_0 + 2s_1 &= 0.5K = -2 \\
K &= -4
\end{aligned}$$

Substituting in the s_0 equation,

$$\begin{aligned}
-4(1.5 + 0.1/\tau_i) &= 2.5 \\
\tau_i &= -0.4/8.5
\end{aligned}$$

Note that some of the tuning parameters are negative. In this approach, there is no guarantee of getting positive parameters, as these are calculated by solving equations.

3. (a) Systems with smaller delay/dead time are easier to handle, compared to those with larger delays.

   (b) Calculate y^* from the block diagram

      i. $y^* = z^{-k}\frac{B}{A}u + (1-z^{-k})\frac{B}{A}u = \frac{B}{A}u$. Thus, y^* gives a k step ahead prediction of y

      ii. The above approach will not work if noise ξ were not zero

   (c) The plant model is,

$$y = z^{-k}\frac{B}{A}u + \frac{C}{A}\xi$$

      i. The prediction model is

$$\hat{y}(n+k|n) = \frac{EB}{C}u(n) + \frac{F}{C}y(n)$$

      Thus, one choice for K is F/C.

      ii. For this choice of K,

$$\begin{aligned} y^{**} &= \frac{EB}{C}u + \frac{F}{C}y \\ &= \frac{EB}{C} + \frac{F}{C}\left[z^{-k}\frac{B}{A}u + \frac{C}{A}\xi\right] \\ &= \frac{EB}{C}u + z^{-k}\frac{BF}{AC}u + \frac{F}{A}\xi \\ &= \frac{B}{C}\left(E + z^{-k}\frac{F}{A}\right)u + \frac{F}{A}\xi \\ &= \frac{B}{C}\frac{C}{A}u + \frac{F}{A}\xi \\ &= \frac{B}{A}u + \frac{F}{A}\xi \end{aligned}$$

      Now, let us try to get a physical interpretation of this model. Substituting $C/A = E + z^{-k}F/A$ in the ARMAX model, we obtain,

$$y = z^{-k}\frac{B}{A}u + E\xi + \frac{F}{A}z^{-k}\xi$$

      The prediction model is obtained by zeroing future noise terms. We obtain,

$$\hat{y} = z^{-k}\frac{B}{A}u + \frac{F}{A}z^{-k}\xi = (\frac{B}{A}u + \frac{F}{A}\xi)z^{-k}$$

      Thus, y^{**} is the k step ahead predictor of $\hat{y}$. Thus, this predictor takes care of the task that cannot be handled by the Smith predictor.

      iii. The main calculation is the solution to Aryabhatta identity