

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>I</b>	<b>Modelling</b>	<b>3</b>
<b>2</b>	<b>Sampled Data System</b>	<b>5</b>
2.1	Disparate Systems in a Loop . . . . .	5
2.2	Establishing Connections . . . . .	6
2.2.1	Continuous Devices to Digital Systems - A/D Converter . . . . .	6
2.2.2	Digital Devices to Continuous Systems - D/A Converter . . . . .	8
2.3	Models of Continuous Time Systems . . . . .	9
2.3.1	Magnetically Suspended Ball . . . . .	9
2.3.2	DC Motor . . . . .	12
2.3.3	Liquid Flow Systems . . . . .	12
2.3.4	van de Vusse Reactor . . . . .	13
2.4	Discretization of Continuous Time Systems . . . . .	14
2.4.1	Solution to the State Space Equation . . . . .	14
2.4.2	Zero Order Hold Equivalent of the State Space Equation . . . . .	16
2.4.3	An Approximate Method of Discretization . . . . .	19
2.4.4	Discretization of Systems with Delay . . . . .	20
2.5	Models of Naturally Discrete Time Systems . . . . .	22
2.5.1	IBM Lotus Domino Server . . . . .	22
2.5.2	Supply Chain Control . . . . .	23
2.6	Connecting all together . . . . .	24
2.6.1	Input-Output View . . . . .	24
2.6.2	Approaches to Controller Design and Testing . . . . .	25
2.7	Matlab Code . . . . .	26
2.8	Problems . . . . .	27
<b>II</b>	<b>Signal Processing</b>	<b>31</b>
<b>3</b>	<b>Linear System</b>	<b>33</b>
3.1	Basic Concepts . . . . .	33
3.1.1	Linearity . . . . .	33
3.1.2	Time Invariance . . . . .	37
3.1.3	Causality and Initial Rest . . . . .	38
3.2	Basic Discrete Time Signals and their Properties . . . . .	39
3.2.1	Representation of a sequence using unit impulse sequence . . . . .	39
3.3	Input-Output Convolution Models . . . . .	40
3.3.1	Input-Output Linearity . . . . .	40

3.3.2	Impulse Response Models . . . . .	42
3.3.3	Relation Between Impulse Response and Step Response Models	45
3.3.4	Impulse Response of Causal Systems . . . . .	46
3.3.5	Parametric and Nonparametric Models . . . . .	47
3.3.6	Recursive Solution to State Space Equations . . . . .	47
3.4	Properties of Convolution . . . . .	48
3.5	External Stability of LTI Systems . . . . .	50
3.6	Matlab Code . . . . .	51
3.7	Problems . . . . .	53
<b>4</b>	<b>Z-Transform</b>	<b>55</b>
4.1	Motivation . . . . .	55
4.2	Absolute Convergence . . . . .	57
4.3	Region of Convergence . . . . .	60
4.3.1	Properties of Region of Convergence . . . . .	63
4.4	Z-Transform Theorems and Examples . . . . .	66
4.4.1	Linearity . . . . .	66
4.4.2	Shifting . . . . .	69
4.4.3	Effect of Damping . . . . .	70
4.4.4	Initial value theorem for causal signal . . . . .	70
4.4.5	Final value theorem for causal signals . . . . .	70
4.4.6	Convolution . . . . .	72
4.4.7	Differentiation . . . . .	73
4.4.8	Z-Transform of Folded or Time Reversed Functions . . . . .	75
4.5	Transfer Function . . . . .	75
4.5.1	Z-transform of Discrete Time State Space Systems . . . . .	76
4.5.2	Gain of a Transfer Function . . . . .	77
4.5.3	Transfer Function of Connected Systems . . . . .	77
4.5.4	Jury's Stability Rule . . . . .	79
4.6	Inverse of Z-transform . . . . .	80
4.6.1	Contour Integration . . . . .	81
4.6.2	Partial Fraction Expansion . . . . .	83
4.6.3	Realization . . . . .	91
4.7	Matlab Code . . . . .	92
4.8	Problems . . . . .	95
<b>5</b>	<b>Frequency Domain Analysis</b>	<b>101</b>
5.1	Basics . . . . .	101
5.1.1	Oscillatory nature of system response . . . . .	101
5.1.2	Continuous and Discrete Time Sinusoidal Signals . . . . .	103
5.1.3	Sampling of Analog Signals . . . . .	107
5.2	Fourier Series and Fourier Transforms . . . . .	109
5.2.1	Fourier Series for Continuous Time Periodic Signals . . . . .	109
5.2.2	Fourier Transform of Continuous Time Aperiodic Signals . . . . .	110
5.2.3	Frequency Response . . . . .	113
5.2.4	Fourier Transform of Discrete Time Periodic Signal . . . . .	114
5.2.5	Fourier Transform of Discrete Time Aperiodic Signals . . . . .	115
5.2.6	Properties of Fourier Transforms of Real Discrete Time Signals	116
5.2.7	Parseval's Theorem . . . . .	120
5.3	Sampling and Reconstruction . . . . .	121
5.3.1	Reconstruction of Analog Signal from Samples . . . . .	124
5.3.2	Sampling Theorem . . . . .	126

5.3.3	Zero Order Hold . . . . .	126
5.4	Filtering . . . . .	127
5.4.1	Pole-Zero Location based Filter Design . . . . .	129
5.4.2	Classification of Filters by Phase . . . . .	132
5.4.3	Lead-Lag Filters . . . . .	132
5.5	Discrete Fourier Transform . . . . .	132
5.6	Matlab Code . . . . .	135
5.7	Problems . . . . .	137
<b>III</b>	<b>Identification</b>	<b>145</b>
<b>6</b>	<b>Identification</b>	<b>147</b>
6.1	Introduction . . . . .	148
6.2	Least Squares Estimation . . . . .	150
6.2.1	Regression Equation for Least Squares Estimation . . . . .	150
6.2.2	Least Squares Problem: Formulation and Solution . . . . .	151
6.3	Time Series . . . . .	154
6.3.1	Covariance in Stationary, Ergodic Processes . . . . .	154
6.3.2	White Noise . . . . .	157
6.3.3	Detection of Periodicity through ACF . . . . .	159
6.3.4	Detection of Transmission Delays . . . . .	160
6.3.5	Covariance of Zero Mean Processes through Convolution . . . . .	163
6.4	Auto Regressive Moving Average (ARMA) Processes . . . . .	164
6.4.1	ARMA, MA, AR Processes . . . . .	164
6.4.2	Determination of Order of MA Processes . . . . .	165
6.4.3	Determination of Order of AR Processes . . . . .	168
6.4.4	Determination of Order of ARMA Processes . . . . .	171
6.5	Prediction Error Models . . . . .	173
6.5.1	Mixed Notation . . . . .	174
6.5.2	One Step Ahead Prediction Error Model . . . . .	175
6.5.3	Classification of Prediction Error Models . . . . .	177
6.5.4	Finite Impulse Response (FIR) Model . . . . .	178
6.5.5	Auto Regressive with eXogeneous input (ARX) model . . . . .	179
6.5.6	Auto Regressive Moving Average with eXogeneous input (AR-MAX) model . . . . .	184
6.5.7	Auto Regressive Integrated Moving Average Model with eXogenous input (ARIMAX) . . . . .	187
6.5.8	Output Error Model . . . . .	188
6.5.9	Box Jenkins (BJ) Model . . . . .	190
6.6	Statistical Properties of Parameter Estimates . . . . .	192
6.6.1	When is Least Squares Estimate Unbiased? . . . . .	193
6.6.2	Best Linear Unbiased Estimate . . . . .	195
6.7	Frequency Domain Interpretation . . . . .	195
6.7.1	Covariance Between Signals of LTI Systems . . . . .	195
6.7.2	Frequency Response of LTI Systems Excited by White Noise	197
6.7.3	Parametric Models . . . . .	198
6.8	Case Study . . . . .	201
6.8.1	Drifting Noise Model . . . . .	201
6.8.2	Identification of an Experimental Heating Tank . . . . .	207
6.9	Maximum Likelihood Estimation . . . . .	212
6.10	Matlab Code . . . . .	216

6.11 Problems . . . . .	228
-------------------------	-----

## IV Transfer Function Approach to Controller Design **233**

<b>7 Structures, Specifications and Tools</b>	<b>235</b>
7.1 General Control Structures . . . . .	235
7.1.1 Feedforward Controller . . . . .	235
7.1.2 One Degree of Freedom Feedback Controller (1-DOF) . . . . .	236
7.1.3 Two Degrees of Freedom Feedback Controller (2-DOF) . . . . .	237
7.2 Proportional Control . . . . .	239
7.2.1 Nyquist Plot for Control Design . . . . .	241
7.2.2 Stability Margins . . . . .	245
7.3 Other Popular Controllers . . . . .	247
7.3.1 Lead Lag Control . . . . .	247
7.3.2 Proportional, Integral, Derivative Control . . . . .	253
7.4 Internal Stability and Realizability . . . . .	257
7.4.1 Forbid Unstable Pole-Zero Cancellation . . . . .	257
7.4.2 Internal stability . . . . .	261
7.4.3 Internal Stability and Controller Realizability . . . . .	263
7.4.4 Closed Loop Delay Specification and Realizability . . . . .	264
7.5 Internal Model Principle and System Type . . . . .	265
7.5.1 Internal Model Principle . . . . .	265
7.5.2 System Type . . . . .	267
7.6 Introduction to Limits of Performance . . . . .	269
7.6.1 Time Domain Limits . . . . .	269
7.6.2 Sensitivity Functions . . . . .	273
7.6.3 Frequency Domain Limits . . . . .	274
7.7 Well behaved signals . . . . .	276
7.7.1 Small Rise Time in Response . . . . .	277
7.7.2 Small Overshoot in Response . . . . .	277
7.7.3 Large decay ratio . . . . .	278
7.8 Solving Polynomial Equations . . . . .	280
7.9 Matlab Code . . . . .	280
7.10 Problems . . . . .	283
<b>8 Proportional, Integral, Derivative Controllers</b>	<b>285</b>
8.1 Sampling Revisited . . . . .	285
8.2 Discretization Techniques . . . . .	286
8.2.1 Area Based Approximation . . . . .	286
8.2.2 Step Response Equivalence Approximation . . . . .	288
8.3 PID Controllers . . . . .	292
8.3.1 Basic Design . . . . .	292
8.3.2 Ziegler-Nichols Method of Tuning . . . . .	293
8.3.3 2-DOF Controller with Integral Action at Steady State . . . . .	294
8.3.4 Bumpless PID Controller with $T_c = S_c$ . . . . .	296
8.3.5 PID Controller with Filtering and $T_c = S_c$ . . . . .	297
8.3.6 2-DOF PID Controller with $T_c = S_c(1)$ . . . . .	300
8.3.7 2-DOF PID Controller with $T_c(1) = S_c(1)$ . . . . .	303
8.4 Matlab Code . . . . .	305
8.5 Problems . . . . .	306

<b>9 Pole Placement Controllers</b>	<b>309</b>
9.1 Dead-Beat and Dahlin Control . . . . .	309
9.2 Pole Placement Controller with Performance Specifications . . . . .	311
9.3 Implementation of Unstable Controllers . . . . .	316
9.4 Internal Model Principle to Improve Robustness . . . . .	318
9.5 Redefining Good and Bad Polynomials . . . . .	323
9.5.1 Summary of Pole Placement Design Procedure . . . . .	327
9.6 Anti Windup Compensator . . . . .	327
9.7 PID Tuning through Pole Placement . . . . .	333
9.8 Matlab Code . . . . .	339
9.9 Problems . . . . .	348
<b>10 Special Cases of Pole Placement Control</b>	<b>351</b>
10.1 Smith Predictor . . . . .	351
10.2 Internal Model Control . . . . .	351
10.2.1 IMC - A Motivation . . . . .	351
10.2.2 IMC Design for Stable Plants . . . . .	353
10.2.3 IMC in Conventional Form for Stable Plants . . . . .	358
10.2.4 PID Tuning through IMC . . . . .	361
10.2.5 IMC Design for Unstable Plants . . . . .	362
10.3 LQR through Pole Placement Control . . . . .	364
10.4 Repetitive Control . . . . .	364
10.5 Matlab Code . . . . .	364
10.6 Problems . . . . .	368
<b>11 Minimum Variance Control</b>	<b>369</b>
11.1 <i>k</i> -Step Ahead Prediction Error Model . . . . .	369
11.1.1 ARMAX Model . . . . .	369
11.1.2 Integrated White Noise Model . . . . .	373
11.1.3 ARIMAX Model . . . . .	375
11.2 Minimum Variance Controller . . . . .	376
11.2.1 ARMAX Model . . . . .	376
11.2.2 Control law for nonminimum phase systems . . . . .	378
11.2.3 Minimum Variance Control Law for ARIMAX Model . . . . .	380
11.3 Generalized Minimum Variance Controller . . . . .	382
11.3.1 ARMAX Model . . . . .	382
11.3.2 ARIMAX Model . . . . .	383
11.3.3 PID Tuning through GMVC . . . . .	385
11.4 Matlab Code . . . . .	389
11.5 Problems . . . . .	395
<b>12 Model Predictive Control</b>	<b>397</b>
12.1 Generalized Predictive Control . . . . .	398
12.1.1 Integrated White Noise Model . . . . .	398
12.1.2 ARIMAX Model . . . . .	403
12.2 $\gamma$ -GPC . . . . .	406
12.2.1 Model Derivation . . . . .	406
12.2.2 Optimization of Objective Function . . . . .	408
12.2.3 Predictive PID, Tuned with Gamma GPC . . . . .	410
12.3 Dynamic Matrix Control . . . . .	412
12.4 Matlab Code . . . . .	415
12.5 Problems . . . . .	419

<b>13 Linear Quadratic Gaussian Control</b>	<b>421</b>
13.1 Spectral Factorization . . . . .	421
13.2 Controller Design . . . . .	424
13.3 Simplified LQG Control Design . . . . .	432
13.4 Introduction to Performance Analysis of Controllers . . . . .	433
13.5 Matlab Code . . . . .	436
13.6 Problems . . . . .	439
<b>V State Space Approach to Controller Design</b>	<b>441</b>
<b>14 State Space Techniques to Controller Design</b>	<b>443</b>
14.1 Pole placement . . . . .	443
14.1.1 Ackermann's formula . . . . .	446
14.1.2 Control Law when System is not in Canonical Form . . . . .	447
14.1.3 Controllability . . . . .	451
14.2 Estimators . . . . .	453
14.2.1 Prediction Estimators . . . . .	454
14.2.2 Observability . . . . .	457
14.2.3 Current Estimators . . . . .	457
14.3 Regulator Design - Combined Control Law and Estimator . . . . .	458
14.4 Linear Quadratic Regulator . . . . .	461
14.4.1 Formulation of Optimal Control Problem . . . . .	462
14.4.2 Solution to Optimal Control Problem . . . . .	463
14.4.3 Infinite Horizon Solution to LQR Design . . . . .	466
14.5 Kalman Filter . . . . .	467
14.6 Matlab Code . . . . .	471
14.7 Problems . . . . .	474
<b>VI Appendices</b>	<b>477</b>
<b>A Mathematical Relations</b>	<b>479</b>
A.1 Differentiation of a quadratic form . . . . .	479
A.1.1 Differentiation with respect to a scalar . . . . .	479
A.1.2 Differentiation with respect to a vector . . . . .	480
A.2 Problems . . . . .	480
<b>B Aryabhatta's Identity</b>	<b>483</b>
B.1 Euclid's Algorithm for GCD of Two Polynomials . . . . .	483
B.2 Algorithms to Solve Aryabhatta's Identity . . . . .	485
B.2.1 Left coprime factorization . . . . .	486
B.2.2 Aryabhatta's Identity . . . . .	488
B.2.3 Solution to Polynomial Equation . . . . .	489
B.3 Matlab Code . . . . .	490
B.4 Problems . . . . .	492
<b>C Installation and Use of Software</b>	<b>493</b>
C.1 SimulinkCode for Verification . . . . .	493

# List of Matlab Code

2.1	Matrix exponential . . . . .	26
2.2	ZOH equivalent state space system . . . . .	26
3.1	Energy of a signal . . . . .	51
3.2	Convolution of two sequences . . . . .	52
4.1	To produce $a^n 1(n)$ . . . . .	92
4.2	To produce $-a^n 1(-n - 1)$ . . . . .	93
4.3	To produce the pole-zero plot in Fig. 4.4 . . . . .	93
4.4	Discrete transfer function of a continuous ss system . . . . .	93
4.5	Stability of a supply chain transfer function . . . . .	93
5.1	Sinusoidal plots for increasing frequency . . . . .	135
5.2	Bode plots for Example 5.7 . . . . .	135
5.3	Bode plot of the moving average filter . . . . .	135
5.4	To produce Bode plot of differencing filter . . . . .	135
6.1	Least square solution of the simple problem discussed in Example 6.4	216
6.2	ACF calculation . . . . .	217
6.3	To demonstrate the periodicity property of ACF . . . . .	217
6.4	To demonstrate the maximum property of ACF at zero lag . . . . .	217
6.5	Determination of order of MA( $q$ ) process . . . . .	218
6.6	Procedure to plot ACF . . . . .	218
6.7	Determination of order of AR( $p$ ) process . . . . .	219
6.8	Calculation of PACF and plotting it . . . . .	219
6.9	Construction of square matrix required to solve PACF $a_{jj}$ . . . . .	220
6.10	Implementation of trial and error procedure to determine ARMA(1,1) process . . . . .	220
6.11	Determination of FIR parameters . . . . .	221
6.12	Determination of ARX parameters . . . . .	222
6.13	Determination of ARMAX parameters . . . . .	222
6.14	Determination of OE parameters . . . . .	223
6.15	Determination of BJ parameters . . . . .	224
6.16	Impact of frequency content of input on plant model mismatch . . . . .	224
6.17	Identifying a plant with a drifting noise model . . . . .	225
6.18	Determination and plotting of PDF vs. $y$ . . . . .	226
6.19	Determination and plotting of Likelihood function $L(w)$ vs. $w$ . . . . .	226
7.1	Procedure to draw root locus in Matlab . . . . .	280
7.2	Procedure to draw Nyquist plot in Matlab . . . . .	280
7.3	Procedure to draw Bode plot . . . . .	281
7.4	A procedure to design lead controllers . . . . .	281

7.5	Bode plot of a lead controller . . . . .	281
7.6	Verification of performance of lead controller on antenna system . . . . .	282
8.1	Continuous to discrete time transfer function . . . . .	305
9.1	Pole placement controller for magnetically suspended ball problem . . . . .	339
9.2	Discretization of continuous transfer function . . . . .	340
9.3	Procedure to split a polynomial into good and bad factors . . . . .	340
9.4	Calculation of desired closed loop characteristic polynomial . . . . .	340
9.5	2-DOF Pole placement controller . . . . .	341
9.6	Simulation of closed loop system with unstable controller . . . . .	341
9.7	Pole placement controller using internal model principle . . . . .	342
9.8	Pole placement controller with internal model of a step for the magnetically suspended ball problem . . . . .	342
9.9	Pole placement controller for motor problem . . . . .	343
9.10	Procedure to split a polynomial into good and bad factors . . . . .	343
9.11	Anti windup control of IBM lotus domino server . . . . .	344
9.12	Demonstration of usefulness of negative PID parameters . . . . .	345
9.13	Solution to Aryabhatta identity arising in PID controller design . . . . .	345
9.14	DC Motor with PD control, tuned through pole placement technique . . . . .	346
9.15	PD control law from polynomial coefficients . . . . .	346
10.1	Splitting a polynomial $B(z^{-1})$ into $B^g$ , $B^-$ and $B^{\text{nm}+}$ . . . . .	364
10.2	Design of internal model controller, $G_Q$ . . . . .	365
10.3	IMC design for viscosity control problem . . . . .	365
10.4	IMC design for the control of van de Vusse reactor . . . . .	365
10.5	IMC design for Lewin's example . . . . .	366
10.6	Design of conventional controller $G_D$ which is an equivalent of internal model controller, $G_Q$ . . . . .	366
10.7	Design of conventional controller $G_D$ for van de Vusse reactor problem . . . . .	367
10.8	IMC design for plants with one unstable pole . . . . .	367
10.9	IMC design for magnetically suspended ball problem . . . . .	368
11.1	Recursive computation of $E_j$ and $F_j$ . . . . .	389
11.2	Recursive computation of $E_j$ and $F_j$ for the system presented in Example 11.2 . . . . .	391
11.3	Solution of Aryabhatta identity Eq. 11.11, discussed in Example 11.3 . . . . .	391
11.4	MacGregor's first example, discussed in Example 11.4 . . . . .	391
11.5	Minimum variance control law design . . . . .	391
11.6	Closed loop transfer functions . . . . .	391
11.7	Cancellation of common factors and determination of covariance . . . . .	392
11.8	Minimum variance control for nonminimum phase systems . . . . .	392
11.9	Minimum variance control for nonminimum phase example of [3] . . . . .	393
11.10	MacGregor's viscosity control problem . . . . .	393
11.11	Splitting a polynomial into good and bad factors . . . . .	393
11.12	GMV Design . . . . .	394
11.13	GMV control design of MacGregor's first example . . . . .	394
11.14	GMV control design of viscosity . . . . .	394
11.15	Example to demonstrate PID tuning through GMVC law . . . . .	394
11.16	PID tuning through GMVC law . . . . .	395
12.1	Model derivation for GPC design in Example 12.1 . . . . .	415
12.2	GPC design in Example 12.2 . . . . .	416

12.3 GPC design for viscosity control in Example 12.3 . . . . .	416
12.4 Miller's example PID controller, tuned with GPC . . . . .	417
12.5 Predictive PID, tuned with GPC . . . . .	417
13.1 Spectral Factorization, see Example 13.3 . . . . .	436
13.2 LQG control design by polynomial method . . . . .	437
13.3 LQG design in Example 13.4 . . . . .	437
13.4 LQG design for viscosity control in Example 13.5 . . . . .	438
13.5 Simplified LQG design . . . . .	438
13.6 LQG design in Example 13.6 . . . . .	438
13.7 Performance curve for LQG control design of viscosity problem . . . . .	439
13.8 Performance curve for GMV control design of MacGregor's first example	439
14.1 Model of inverted pendulum . . . . .	471
14.2 Pole placement controller for inverted pendulum . . . . .	471
14.3 Compensator calculation for Example 14.7 . . . . .	472
14.4 Kalman filter example of estimating a constant . . . . .	472
B.1 Left Coprime Factorization . . . . .	490
B.2 Aryabhatta Identify . . . . .	491
B.3 Solution to polynomial equation . . . . .	491
B.4 Solution to another polynomial equation . . . . .	491