**Department of Chemical Engineering**
**IIT Bombay**
**CL692, Digital Control**
**Assignment 7**
**Handed out on: 26 Sep 2006**
**To be completed by: 5 Oct 2006**

1. This problem is concerned with determination of where the stable region in the $s$-plane gets mapped to $z$-plane under trapezoidal approximation.

   (a) Show that the trapezoidal approximation, given by Eq. 8.3 is equivalent to

   $$z = \frac{1 + \frac{sT_s}{2}}{1 - \frac{sT_c}{2}}$$

   (b) Find out where the left half of the $s$ plane will be mapped in the $z$ plane using the above transformation (Hint: substitute $s = a + jb$ in the above equation for $a < 0$ and find out what $z$ you get. Repeat this for $a = 0$.) Does this agree with the notion of the $z$ domain stability region discussed earlier?

2. This question is concerned with discretization of the PID controller given in Eq. 8.59, where, $0 < b < 1$ and $N$ is of the order of 10. As usual, $R$, $Y$ and $U$, respectively, refer to setpoint, output and input.

   (a) Use trapezoidal approximation for both derivative and integral terms, *i.e.,* substitute for $s$ as

   $$s \leftrightarrow \frac{2}{T_s}\frac{z-1}{z+1}$$

   and arrive at the following result:

   $$U(z) = K\left[b + b_i\frac{z+1}{z-1}\right]R(z) - K\left[1 + b_i\frac{z+1}{z-1} + \frac{(z-1)b_d}{z-a_d}\right]Y(z)$$

   where,

   $$b_i = \frac{T_s}{2\tau_i}, \quad b_d = \frac{2N\tau_d}{2\tau_d + NT_s}, \quad a_d = \frac{2\tau_d - NT_s}{2\tau_d + NT_s}$$

   (b) Simplify the above expressions to arrive at a controller in the usual $R_c$, $S_c$, $T_c$ form with $S_c \neq T_c$:

   $$\begin{aligned}(1 - z^{-1})(1 - a_d z^{-1})U(z) &= [t_0 + t_1 z^{-1} + t_2 z^{-2}]R(z) \\ &\quad - [s_0 + s_1 z^{-1} + s_2 z^{-2}]Y(z)\end{aligned}$$

   where,

   $$\begin{aligned}t_0 &= K(b + b_i) \\ t_1 &= -K(b(1 + a_d) - b_i(1 - a_d)) \\ t_2 &= Ka_d(b - b_i) \\ s_0 &= K(1 + b_i + b_d) \\ s_1 &= -K(1 + a_d + 2b_d - b_i(1 - a_d)) \\ s_2 &= K(a_d + b_d - b_i a_d)\end{aligned}$$

   (c) Check that $T_c(1) = S_c(1) = 2Kb_i(1 - a_d)$ and hence that this controller satisfies the condition required for the plant output to track the setpoint.

3. This is an open ended question. Please submit it in a separate sheet. In Chapter 8 of the Text, we have presented six different discrete PID controllers. Suggest a method to compare them, either analytically, or through simulation, or both. If you carry out simulations using simulink, to the extent possible, use the simulink programs presented in the Text.