Department of Chemical Engineering
IIT Bombay
CL692, Digital Control
Assignment 7
Handed out on: 19 Sep 2006
To be completed by: 25 Sep 2006

1. In the Text, we designed a proportional controller for the plant $H = \frac{1}{z(z-1)}$ using a *particular* contour C_1 that encircled *both* the poles. In this problem you indent the contour in such a way that the pole at 1 is *excluded*, as in Fig. 1. Using the Nyquist

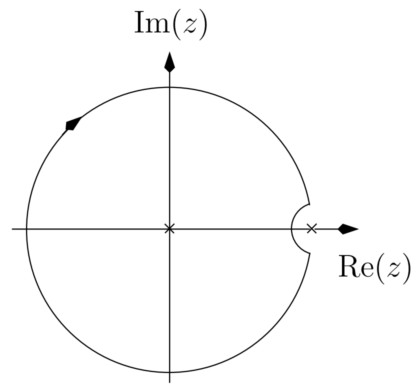
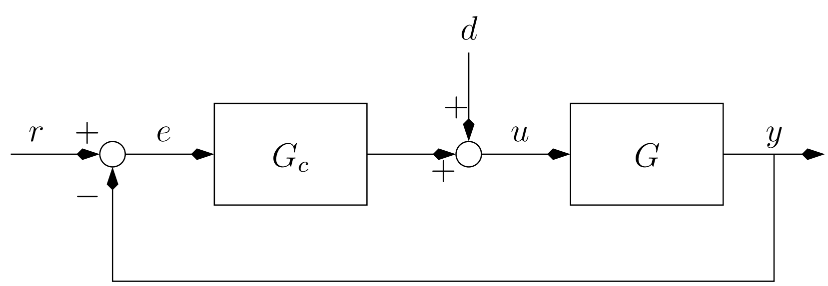


Figure 1: An alternative contour C_1

plot for this C_1, determine the range of proportional controller for which the closed loop system is stable.

2. This problem is concerned with the demonstration that internal stability of the closed loop system shown below can be expressed in terms of a few conditions on the sensitivity function (to be described below).

   (a) Write down the 2×2 transfer function between $\begin{bmatrix} r \\ d \end{bmatrix}$ (treated as input) and $\begin{bmatrix} e \\ u \end{bmatrix}$ (treated as output) in the following figure. Hint: see Eq. 8.12 in the Text.



   (b) State the internal stability condition, in terms of the stability of the entries of 2×2 transfer function matrix, obtained in (a)?

   (c) State the internal stability condition in terms of the unstable pole-zero cancellation and external stability of the closed loop system?

(d) Let the sensitivity function S be defined as,

$$S = \frac{1}{1 + GG_c}$$

Show that the unstable poles of G will not be cancelled by the zeros of G_c, if $S(p_i) = 0$, $n \geq i \geq 1$, where, p_i are the unstable poles of G.

(e) Show that the nonminimum phase zeros of $G(z)$ will not be cancelled by the unstable poles of G_c, if $S(z_j) = 1$, $m \geq j \geq 1$, where, z_j are the nonminimum phase zeros of G.

(f) State the internal stability condition in terms of conditions on S, derived in (d) and (e), and stability of S?

By the way, (f) is known as the interpolation condition on the sensitivity function, using which, controllers can be designed.

3. Consider a closed loop system that includes a controller so that the error to a unit step change in the reference signal behaves as $r^n \cos \omega n$. Show that if the steady state error of this system to a ramp of unit slope has to be less than a small value, say, μ, the following relation should be satisfied:

$$T_s \frac{1 - r\cos\omega}{1 - 2r\cos\omega + r^2} < \mu \tag{1}$$