Department of Chemical Engineering
IIT Bombay
CL692, Digital Control
Assignment 5
Handed out on: 29 Aug 2006
To be completed by: 4 Sep 2006

1. This problem demonstrates that care should be taken while calculating the means in the time delay detection problem.

   (a) Using `xcov` function of Matlab, determine the CCF, *i.e.*, $r_{uy}(n)$, between two signals $\{u(n)\} = \{1, 2\}$ and the delayed signal $\{y(n)\} = \{0, 1, 2\}$. Do these results agree with hand calculations? Explain.

   (b) Repeat the above steps with $\{u(n)\} = \{-0.5, 0.5\}$ and $\{y(n)\} = \{0, -0.5, 0.5\}$. What do you observe now? Why?

2. Consider the system

$$y(n) + a_1 y(n-1) = b_1 u(n-1) + \xi(n) + c_1 \xi(n-1)$$

with

$$\begin{aligned} \gamma_{uu}(l) &= \delta(l)\sigma_u^2 \\ \gamma_{\xi\xi}(l) &= \delta(l)\sigma_\xi^2 \end{aligned}$$

and u, ξ uncorrelated. Show that

$$\begin{aligned} \gamma_{y\xi}(k) + a_1\gamma_{y\xi}(k-1) &= \gamma_{\xi\xi}(k) + c_1\gamma_{\xi\xi}(k-1) \\ \gamma_{y\xi}(0) &= \sigma_\xi^2 \\ \gamma_{y\xi}(1) &= (c_1 - a_1)\sigma_\xi^2 \\ \gamma_{yu}(k) + a_1\gamma_{yu}(k-1) &= b_1\gamma_{uu}(k-1) \\ \gamma_{yu}(0) &= 0 \\ \gamma_{yu}(1) &= b_1\sigma_u^2 \\ \gamma_{yy}(k) + a_1\gamma_{yy}(k-1) &= b_1\gamma_{yu}(-k+1) + \gamma_{y\xi}(-k) + c_1\gamma_{y\xi}(-k+1) \\ \gamma_{yy}(0) &= \frac{b_1^2\sigma_u^2 + (1 + c_1^2 - 2a_1c_1)\sigma_\xi^2}{1 - a_1^2} \\ \gamma_{yy}(1) &= \frac{-a_1b_1^2\sigma_u^2 + (c_1 - a_1)(1 - a_1c_1)\sigma_\xi^2}{1 - a_1^2} \end{aligned}$$

Hint: You may consider useful to derive these equations in the same order as given.