Department of Chemical Engineering
IIT Bombay
CL692, Digital Control
Assignment 5
Handed out on: 29 Aug 2006
To be completed by: 4 Sep 2006

1. The following code, available at
`HOME/ident/matlab/sys_29.m` carries out the required calculation:

```
1 u = [1 2]; y = [0 1 2];
2 ruy = xcov(u,y);
3 mu = mean(u);
4 u_zm = u - mu;
5 y_zm = y - mu;
6 ruy_zm = xcov(u_zm,y_zm);
```

The first calculation (line 2) gives,

$$\{r_{uy}(n)\} = \texttt{ruy} = \{-0.5, 0.5, 0.5, -0.5, 0\}.$$

This has the shortcoming that there is no one maximum value. On the other hand, in the second calculation, we obtain,

$$\texttt{ruy\_zn} = \{-0.25, 0.5, -0.25, 0, 0\},$$

which is in agreement with what we expect. The reason is that Matlab takes the mean of u as 1.5 and that of y as $1 = (0+1+2)/3$, whereas, we would like to use the same mean for both input and output.

This problem is overcome in the second calculation (line 6), in which, we use only a zero mean signal as the input. As a result, the output is a shifted version of the input, with both the signals being of zero mean.

To summarize, we should use zero mean signals for input, or carry out the mean calculations in a consistent manner.

2. Multiply Eq. **??** by $\xi(n-k)$ and take expectation:

$$\gamma_{y\xi}(k) + a_1\gamma_{y\xi}(k-1) = \gamma_{\xi\xi}(k) + c_1\gamma_{\xi\xi}(k-1)$$

With $k = 0$,

$$\gamma_{y\xi}(0) = \sigma_\xi^2$$

With $k = 1$,

$$\begin{aligned}\gamma_{y\xi}(1) + a_1\gamma_{y\xi}(0) &= c_1\sigma_\xi^2 \\ \gamma_{y\xi}(1) &= (c_1 - a_1)\sigma_\xi^2\end{aligned}$$

For $k = 2$,

$$\begin{aligned}\gamma_{y\xi}(2) + a_1\gamma_{y\xi}(1) &= 0 \\ \gamma_{y\xi}(2) &= -a_1\gamma_{y\xi}(1) = -a_1(c_1 - a_1)\sigma_\xi^2 \\ \gamma_{y\xi}(m) &= (-1)^{m-1}a_1^{m-1}(c_1 - a_1)\sigma_\xi^2\end{aligned}$$

Multiply Eq. **??** by $u(n-k)$ and take expectation:

$$\gamma_{yu}(k) + a_1\gamma_{yu}(k-1) = b_1\gamma_{uu}(k-1)$$

With $k=0$,

$$\gamma_{yu}(0) = 0$$

With $k=1$,

$$\begin{aligned}\gamma_{yu}(1) + a_1\gamma_{yu}(0) &= b_1\gamma_{uu}(0) = b_1\sigma_u^2 \\ \gamma_{yu}(1) &= b_1\sigma_u^2\end{aligned}$$

With $k=2$,

$$\begin{aligned}\gamma_{yu}(2) + a_1\gamma_{yu}(1) &= b_1\gamma_{uu}(1) = 0 \\ \gamma_{yu}(2) &= -a_1b_1\sigma_u^2\end{aligned}$$

Multiply Eq. **??** by $y(n-k)$ and take expectation:

$$\gamma_{yy}(k) + a_1\gamma_{yy}(k-1) = b_1\gamma_{yu}(-k+1) + \gamma_{y\xi}(-k) + c_1\gamma_{y\xi}(-k+1)$$

With $k=0$,

$$\begin{aligned}\gamma_{yy}(0) + a_1\gamma_{yy}(1) &= b_1\gamma_{yu}(1) + \gamma_{y\xi}(0) + c_1\gamma_{y\xi}(1) \\ &= b_1^2\sigma_u^2 + \sigma_\xi^2 + c_1(c_1-a_1)\sigma_\xi^2\end{aligned}$$

With $k=1$,

$$\begin{aligned}\gamma_{yy}(1) + a_1\gamma_{yy}(0) &= b_1\gamma_{yu}(0) + c_1\gamma_{ye}(0) = c_1\sigma_\xi^2 \\ \gamma_{yy}(1) &= c_1\sigma_\xi^2 - a_1\gamma_{yy}(0)\end{aligned}$$

Substituting for $\gamma_{yy}(1)$ in the previous expression and solving for $\gamma_{yy}(0)$, we obtain Eq. **??**. Substituting this in the expression for $\gamma_{yy}(1)$ and simplifying, we obtain Eq. **??**.