Department of Chemical Engineering
IIT Bombay
CL692, Digital Control
Assignment 4
Handed out on: 21 Aug 2006
To be completed by: 28 Aug 2005

1. Another approach to frequency response is given in this problem. Consider a system with a transfer function $G(z)$ whose poles are inside the unit circle. Suppose that it is subjected to the input $u(k) = M \cos kw$ where M is the amplitude, w the frequency in rad/sample and k the sample number.

   (a) Show that the Z-transform of the output is given by
   $$Y(z) = \frac{M}{2}\left(\frac{z}{z - e^{j\omega}} + \frac{z}{z - e^{-j\omega}}\right) G(z)$$

   (b) Show that this can be written as
   $$Y(z) = \frac{\alpha z}{z - e^{j\omega}} + \frac{\alpha^* z}{z - e^{-j\omega}} + \sum_{i=1}^{n} \frac{D_i z}{z - p_i}$$
   where α is given by
   $$\alpha = \frac{M}{2} G\left(e^{jw}\right)$$

   (c) Show that the Z-transform of the steady state portion of the output is
   $$\begin{aligned} Y(z) &= \frac{M}{2}\left[\frac{G\left(e^{jw}\right) z}{z - e^{j\omega}} + \frac{G\left(e^{-j\omega}\right) z}{z - e^{-j\omega}}\right] \\ &= \frac{M}{2}\left|G\left(e^{jw}\right)\right|\left[\frac{e^{j\phi} z}{z - e^{j\omega}} + \frac{e^{-j\phi} z}{z - e^{-j\omega}}\right] \end{aligned}$$

   (d) Invert this and show that
   $$\begin{aligned} y(k) &= \frac{M}{2}\left|G\left(e^{jw}\right)\right|\left(e^{j\phi} + \left(e^{jw}\right) e^{-j\phi} e^{-j\omega}\right) \\ &= M\left|G\left(e^{jw}\right)\right| \cos\left(kw + \phi\right) \end{aligned}$$
   This shows that the output also is a sinusoid and shifted in phase by ϕ, the phase angle of $G\left(e^{jw}\right)$ and amplified by $\left|G\left(e^{jw}\right)\right|$.

   (e) Where did you use the fact that all the poles of $G(z)$ are inside the unit circle?

2. An LTI system, initially at rest and with impulse response $g(n)$,
   $$g(n) = \delta(n) - \sqrt{2}\delta(n-1) + \delta(n-2)$$
   is subjected to an input
   $$u(n) = [\cos\frac{\pi}{4}n + \cos\frac{\pi}{2}n]1(n)$$

   (a) Calculate the output by convolution techniques.

   (b) Find the zeros of the transfer function $G(z)$ and using this explain the results of (a).

   (c) Draw a Bode plot of $G(z)$ and using this, explain the results of (a).