Department of Chemical Engineering
IIT Bombay
CL692, Digital Control
Assignment 2
Handed out on: 07 August 2006
To be completed by: 14 Aug 2005

1. Commutativity of i/o linear system depends on both linearity as well as time invariance of both systems. Two examples are provided now.

   (a) Consider two systems A and B where system A is an LTI system with unit sample response $g(n) = \left(\frac{1}{2}\right)^n 1(n)$. System B, on the other hand, is linear but time-varying. Specifically, if the input to system B is $w(n)$, its output $z(n) = nw(n)$. Show that the commutativity property does not hold for these two systems by computing the response of each of the systems below to the input $u(n) = \delta(n)$.

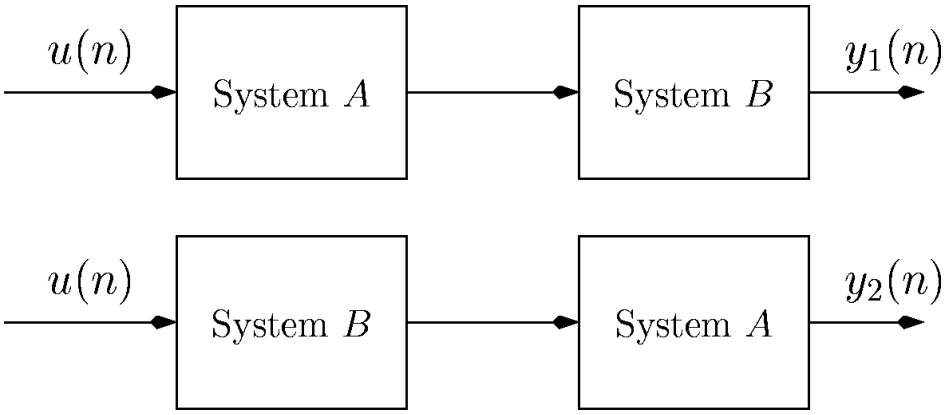

Figure 1: Checking commutativity

   (b) Suppose that we replace system B in each of the interconnected systems of the above Fig. by the system with the following relationship between its input $w(n)$ and output $z(n)$: $z(n) = w(n) + 2$. Repeat the calculations of part (a) in this case.

2. As the impulse response can be constructed from step response, the latter also has all the information about the I/O linear system. The following relation shows that it is possible to calculate the response to an arbitrary input given the step response:

   $$\{y(n)\} = \{s(n)\} * \{\Delta u(n)\}$$

   where, $s(n)$ is the step response and $\Delta u(n) = u(n) - u(n-1)$. Verify whether this is correct. If incorrect, derive the correct relation.

3. Given $s(n) = \left(\frac{1}{2}\right)^n 1(n+1)$ and $u(n) = \left(-\frac{1}{2}\right)^n 1(n)$, find $y(n)$.