Department of Chemical Engineering
IIT Bombay
CL692, Digital Control
Solution to Assignment 2
To be completed by: 14 Aug 2006

1. Commutativity check:

   (a) We will model the situation corresponding to the top figure first. Let $w(n)$ be the output of the first system.

   $$w(n) = \delta(n) * \left(\frac{1}{2}\right)^n 1(n) = \left(\frac{1}{2}\right)^n$$
   $$y_1(n) = nw(n) = n\left(\frac{1}{2}\right)^n 1(n)$$

   Let us now calculate the output for the bottom figure:

   $$w(n) = n\delta(n) = 0$$
   $$\therefore \quad y_2(n) = 0$$

   As $y_1(n) \neq y_2(n)$, Not commutative. Reason: Not time or shift invariant.

   (b) Now we will repeat the calculations for B, described by, $z(n) = w(n) + 2$.

   $$\begin{aligned}
   y_1(n) &= \left(\frac{1}{2}\right)^n 1(n) + 2 \\
   y_2(n) &= [\delta(n) + 2] * \left[\left(\frac{1}{2}\right)^n 1(n)\right] \\
   &= \delta(n) * \left(\frac{1}{2}\right)^n 1(n) + 2 * \left(\frac{1}{2}\right)^n 1(n) \\
   &= \left(\frac{1}{2}\right)^n 1(n) + \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k 1(k).2 \\
   &= \left(\frac{1}{2}\right)^n 1(n) + 2\sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k \\
   &= \left(\frac{1}{2}\right)^n 1(n) + 2.\left(\frac{1}{1-\frac{1}{2}}\right) \\
   &= \left(\frac{1}{2}\right)^n 1(n) + 4
   \end{aligned}$$

   As $y_1(n) \neq y_2(n)$, Not Commutative. Reason: Not Linear.

2. Same as in the book.

3.

$$\begin{aligned}
s(n) &= \left(\frac{1}{2}\right)^n 1(n+1) \\
u(n) &= \left(-\frac{1}{2}\right)^n 1(n) \\
u(n-1) &= \left(-\frac{1}{2}\right)^{n-1} 1(n-1) \\
\Delta u(n) &= \left(-\frac{1}{2}\right)^n 1(n) - \left(-\frac{1}{2}\right)^{n-1} 1(n-1) \\
\Delta u(0) &= 1 \\
\Delta u(n) &= \left(-\frac{1}{2}\right)^n - \left(-\frac{1}{2}\right)^{n-1}, \ n \geq 1 \\
&= \left(-\frac{1}{2}\right)^n - \left(-\frac{1}{2}\right)^n / \left(-\frac{1}{2}\right) = 3\left(-\frac{1}{2}\right)^n, \quad n \geq 1
\end{aligned}$$

Therefore,

$$\begin{aligned}
\Delta u(n) &= \delta(n) + 3\left(-\frac{1}{2}\right)^n 1(n-1) \\
y_2(n) &= s(n) * (\Delta u(n)) = \sum_{k=-\infty}^{\infty} s(k)\Delta u(n-k) \\
\text{Second term} &= 3\sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k 1(k+1)\left(-\frac{1}{2}\right)^{n-k} 1(n-k-1) \\
&= 3\sum_{k=-1}^{\infty} \left(\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)^{n-k} 1(n-k-1) \\
&= 3 \times 1(n) \sum_{k=-1}^{n-1} \left(\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)^{n-k} \\
&= 3 \times 1(n) \left(-\frac{1}{2}\right)^n \sum_{k=-1}^{n-1} (-1)^k \\
\text{First Term} &= \left(\frac{1}{2}\right)^n 1(n+1)
\end{aligned}$$

Summing the two,

$$y(n) = \left(\frac{1}{2}\right)^n 1(n+1) + 3\left(-\frac{1}{2}\right)^n 1(n) \sum_{k=-1}^{n-1} (-1)^k$$