Department of Chemical Engineering
IIT Bombay
CL692, Digital Control
Assignment 1
Handed out on: 31 July 2006
To be completed by: 07 Aug 2006

1. It is desired to model the flow of water into and out of a tank of uniform cross-section, see Fig. 1. The inflow rate $F_i(t)$ need not be steady. The opening of the outflow valve can be adjusted so as to reduce the variations in the outflow. The outflow rate $F(t)$ is a multiple of the height of water in the tank and a variable $x(t)$ that is a function of valve opening, *i.e.*,

$$F(t) = x(t)h(t) \tag{1}$$

   Derive a linearized model of this system in deviation variables.

2. Recall that we derived the ZOH equivalent of

$$\dot{x}(t) = Fx(t) + Gu(t - D)$$

   with

$$D = (d - 1)T_s + D'$$

   and

$$0 < D' \leq T_s$$

   for the case of $d = 1$.

   (a) Derive the expressions for B_0 and B_1, starting from the integral expression.

   (b) Derive the ZOH equivalent discrete state space model for $d = 2$.

   (c) Is there any relation between the matrices B_0 and B_1 that we derived for $d = 1$ and those for $d = 2$?

   (d) Can you repeat the above two for a general integer $d > 0$?

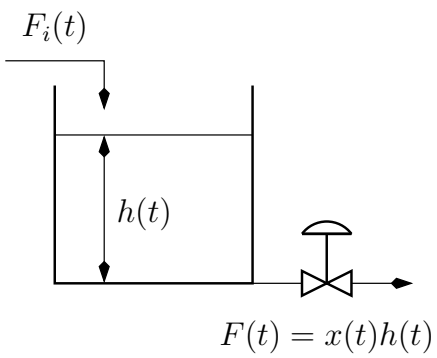

Figure 1: Water flow in a tank