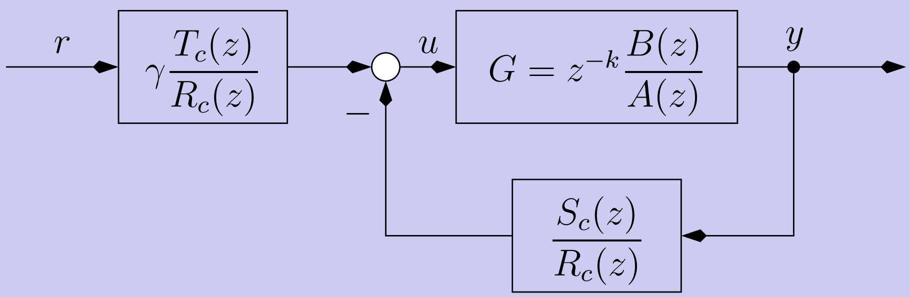
1. Design of Pole Placement Controller: Summary

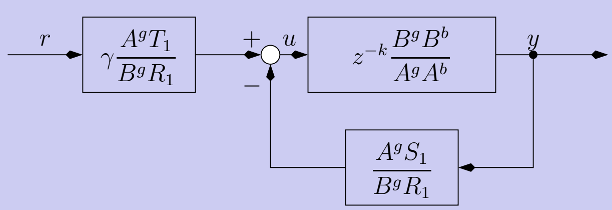
- $y_m(z^{-1}) = \gamma z^{-k} \dfrac{B_r}{\phi_{cl}} r$
- Rise time, overshoot $\Rightarrow$ determine ϕ_{cl}
- $\gamma = \dfrac{\phi_{cl}(1)}{B_r(1)}$
- Controller: $R_c(z)u(n) = \gamma T_c(z) r(n) - S_c(z) y(n)$



- $A = A^g A^b$, $B = B^g B^b$
  $T_c = A^g T_1$, $Rc = B^g R_1$, $S_c = A^g S_1$

2. Design of Pole Placement Controller: Summary

- $A = A^g A^b$, $B = B^g B^b$
  $T_c = A^g T_1$, $Rc = B^g R_1$, $S_c = A^g S_1$
- Solve $A^b R_1 + z^{-k} B^b S_1 = \phi_{cl}$ for R_1, S_1, $T_1 = 1$, $B_r = B^b$



$$T = \gamma \frac{A^g T_1}{B^g R_1} \; \frac{z^{-k}\dfrac{B^g B^b}{A^g A^b}}{1 + z^{-k}\dfrac{B^g B^b}{A^g A^b}\dfrac{A^g S_1}{B^g R_1}} = \gamma \frac{T_1}{R_1} \; \frac{z^{-k}\dfrac{B^b}{A^b}}{1 + z^{-k}\dfrac{B^b S_1}{A^b R_1}}$$

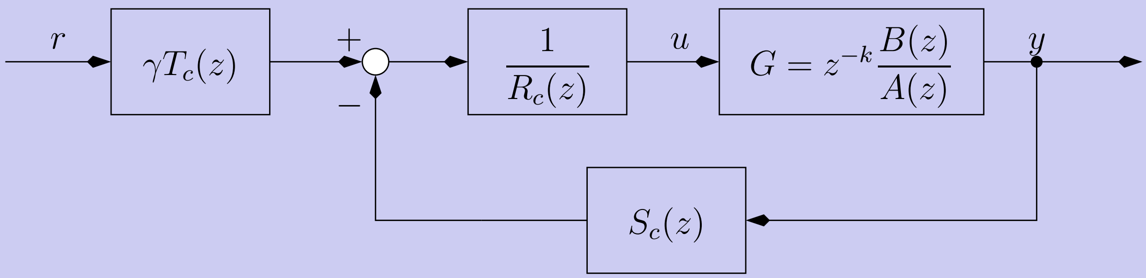
$$= \gamma \frac{T_1}{R_1} \; \frac{z^{-k} B^b R_1}{A^b R_1 + z^{-k} B^b S_1} = \gamma \frac{z^{-k} B^b T_1}{A^b R_1 + z^{-k} B^b S_1} = \gamma z^{-k} \frac{B_r}{\phi_{cl}}$$

3. Unstable Controller

- Recall from the previous slide,

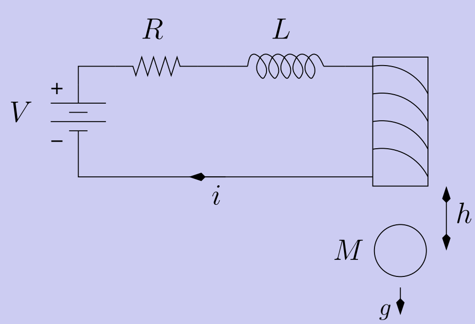
$$T = \gamma \frac{T_1}{R_1} \frac{z^{-k} B^b R_1}{A^b R_1 + z^{-k} B^b S_1} = \gamma \frac{z^{-k} B^b T_1}{A^b R_1 + z^{-k} B^b S_1}$$

- Cancellation of R_1, if unstable, is avoidable
- Take R_c inside the loop



- Example given in the Text

4. Magnetically Suspended Ball



- Current through coil induces magnetic force
- Magnetic force balances gravity
- Ball is suspended in midair - 1 cm from core
- Want to move to another equilibrium

Force balance:

$$M \frac{d^2 h}{dt^2} = Mg - \frac{Ki^2}{h}$$

Voltage balance

$$V = L \frac{di}{dt} + iR$$

5. Magnetically Suspended Ball - Discrete Model

Sample continuous time model using $T_s = 0.01s$. Using myc2d.m,

$$G(z) = z^{-1}\frac{(-3.7209\times10^{-5} - 1.1873\times10^{-4}z^{-1} - 2.2597\times10^{-5}z^{-2})}{1 - 2.4668z^{-1} + 1.7721z^{-2} - 0.3679z^{-3}}$$
$$= z^{-1}\frac{-3.7209\times10^{-5}(1+2.9877z^{-1})(1+0.2033z^{-1})}{(1-1.3678z^{-1})(1-0.7311z^{-1})(1-0.3679z^{-1})} = z^{-k}\frac{B}{A}$$

Split A and B:

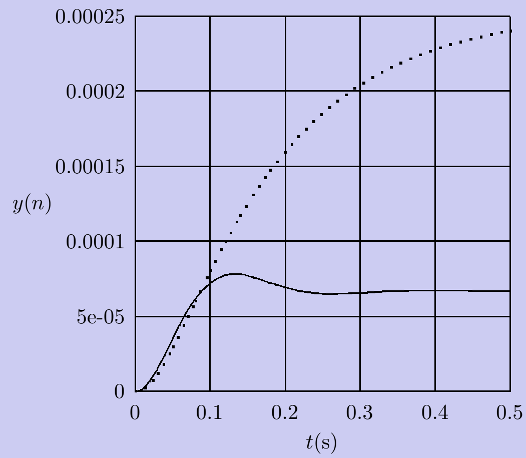
$$A = A^g A^b$$
$$A^g = (1-0.7311z^{-1})(1-0.3679z^{-1})$$
$$A^b = (1-1.3678z^{-1})$$
$$B^g = -3.7209\times10^{-5}(1+0.2033z^{-1})$$
$$B^b = (1+2.9877z^{-1})$$

6. Perturbations of Initial Conditions

- Calculate controller
- Perturb initial conditions
  - x_1: distance between ball and armature
  - $x_1(0)$ perturbed in the range of ± 0.005m
  - x_2: initial velocity, not perturbed
  - x_3: current through circuit
  - $x_3(0)$ perturbed in the range ± 1 amp
- Controller is robust

7. Perturbations to Initial Conditions

- $Y(z) = c(zI - A)^{-1}B$
- For $c = [1\ \ 0\ \ 0]$, design controller
- Perturb c to $[1.1\ \ 0\ \ 0]$, step change of 0.0001



Solid line corresponds to $c = [1.1\ \ 0\ \ 0]$
Dotted line corresponds to $c = [0.9\ \ 0\ \ 0]$

8. Perturbations to Initial Conditions - Why Not Robust?

- Recall the controller

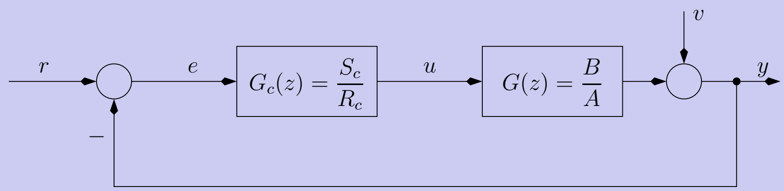
$$R_c(z)u(n) = \gamma T_c(n)r(n) - S_c(z)y(n)$$

- How is γ calculated?

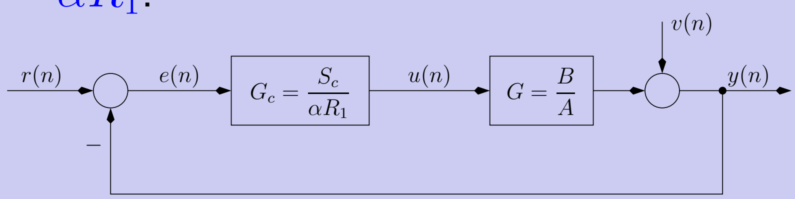
$$\gamma = \frac{\phi_{cl}(1)}{B_r(1)} = \frac{\phi_{cl}(1)}{B^b(1)}$$

- Actual B^b is different from assumed B^b by a factor of 1.1 or 0.9

9. Internal Model Principle



- $\alpha(z^{-1})$ = least common denominator of the unstable poles of $R(z^{-1})$ and of $V(z^{-1})$
- Let there be no common factors between $\alpha(z^{-1})$ and $B(z^{-1})$
- Can find a controller $G_c(z)$ for servo/tracking (following R) and regulation (rejection of disturbance V) if R_c contains α, say, $R_c = \alpha R_1$:



10. Internal Model Principle for Robustness

- Step disturbance has $\Delta = 1 - z^{-1}$ in denominator
- If the feedback loop has Δ in denominator, step disturbance can be rejected
- Recall controller and Aryabhatta identity

$$R_c(z)u(n) = \gamma T_c(z)r(n) - S_c(z)y(n)$$
$$A^b R_1 + z^{-k} B^b S_1 = \phi_{cl}$$

- Make R_c to include Δ as follows. Let $R_c = B^g \Delta R_1$
- Construct Aryabhatta identity

$$A^b \Delta R_1 + z^{-k} B^b S_1 = \phi_{cl}$$

- A^b, Δ, k, B^b, ϕ_{cl} are known
- Aryabhatta identity be solved for R_1 and S_1

11. Controller with Internal Model, Plant Perturbation

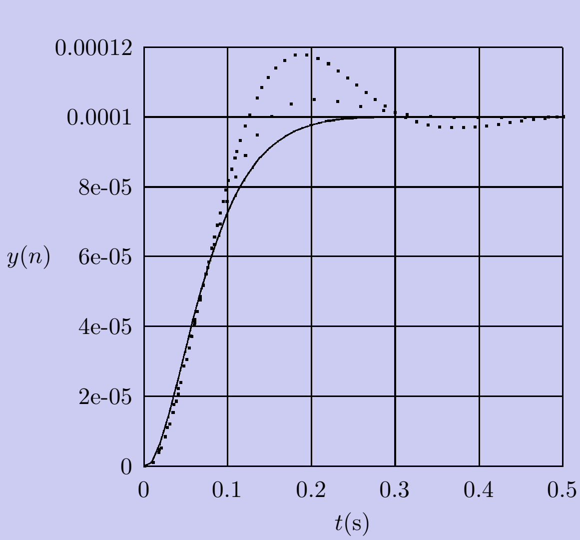
$$A^g = (1 - 0.7311z^{-1})(1 - 0.3679z^{-1})$$
$$A^b = (1 - 1.3678z^{-1})$$
$$B^g = -3.7209 \times 10^{-5}(1 + 0.2033z^{-1})$$
$$B^b = (1 + 2.9877z^{-1})$$
$$\varepsilon = 0.05$$
$$r = 0.8609$$
$$\omega = 0.1571$$
$$\phi_{cl} = 1 - 1.7006z^{-1} + 0.7411z^{-2}$$

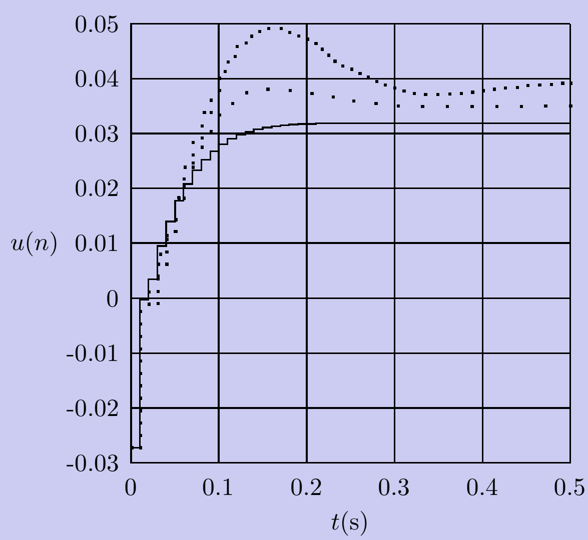
Solve Aryabhatta identity:

$$(1 - 1.3678z^{-1})(1 - z^{-1})R_1 + z^{-1}(1 + 2.9877z^{-1})S_1 = 1 - 1.7006z^{-1} + 0.7411z^{-2}$$

$R_1 = 1 + 0.4507z^{-1}$, $S_1 = 0.2165 - 0.2063z^{-1}$

12. Controller with Internal Model, Plant Perturbation





- Sparse dotted lines: nominal plant;
- Solid lines: $c = \begin{bmatrix} 1.1 & 0 & 0 \end{bmatrix}$;
- Thick dotted lines: $c = \begin{bmatrix} 0.9 & 0 & 0 \end{bmatrix}$;

13. Cancellation of Left Half Plane Poles with Zeros

Control of DC Motor (Astrom & Wittenmark)
Velocity of motor shaft - x_1, position - x_2.

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For a step change in command signal, the pole placement controller should ensure

- rise time of 3 seconds
- overshoot of not more than 0.05

Choose $T_s = 0.25s$

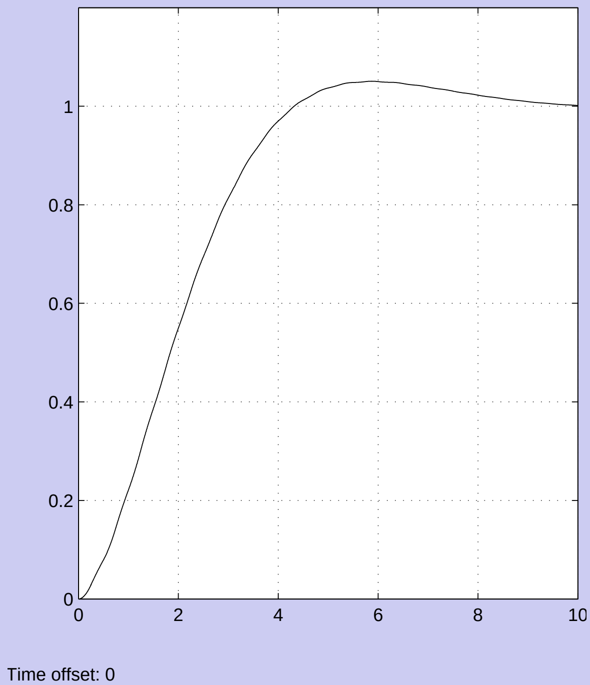
$$G(z) = z^{-1}\frac{0.0288 + 0.0265z^{-1}}{1 - 1.7788z^{-1} + 0.7788z^{-2}}$$

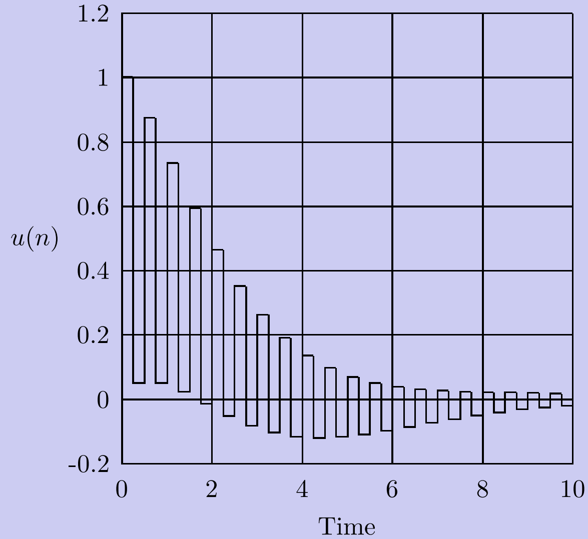
14. Cancellation of Left Half Plane Poles with Zeros

We get the following factorizations:

$$\begin{aligned} A^b &= 1 - z^{-1} \\ A^g &= 1 - 0.7788z^{-1} \\ B^b &= 1 \\ B^g &= 0.0288 + 0.0265 \\ &= 0.0288(1 + 0.9201z^{-1}) \\ \phi_{cl} &= 1 - 1.7502z^{-1} + 0.7791z^{-2} \\ R_1 &= 1 - 0.7791z^{-1} \\ S_1 &= 0.0289 \end{aligned}$$

15. Performance of Pole Placement Controller On DC Motor





Oscillations - due to cancellation of pole-zero in Left Half Plane

16. Why is Control Effort Oscillatory?

Recall controller:

$$R_c(z)u(n) = \gamma T_c(z)r(n) - S_c(z)y(n)$$
$$u(n) = \gamma \frac{T_c(z)}{R_c(z)} r(n) - \frac{S_c(z)}{R_c(z)} y(n)$$

Recall,

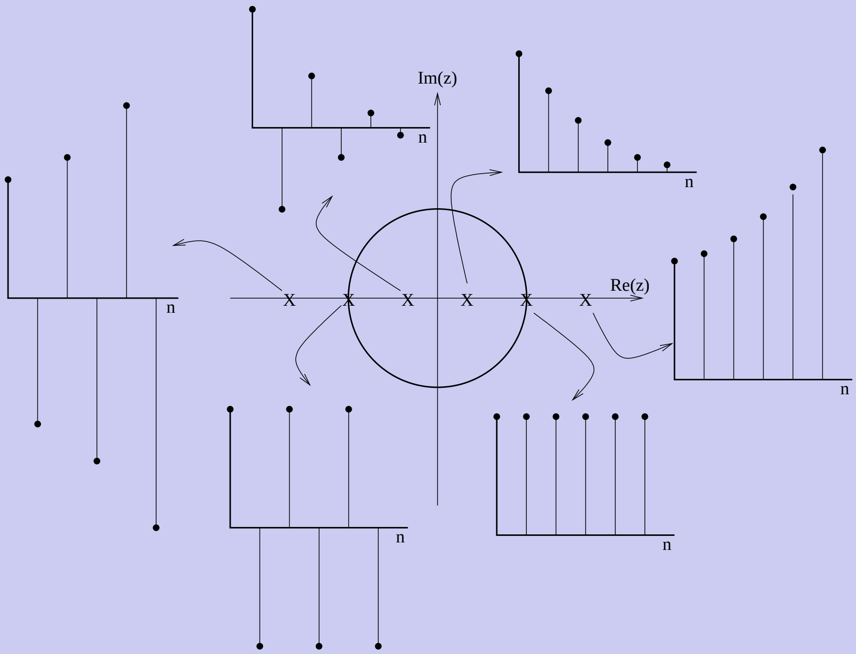
$$R_c = B^g R_1, \quad T_c = A^g T_1, \quad S_c = A^g S_1$$

Substituting, the control law becomes

$$u(n) = \gamma \frac{A^g T_1}{B^g R_1} r(n) - \frac{A^g S_1}{B^g R_1} y(n)$$

If B^g has root with negative real part (e.g. -0.9201), as in the antenna control problem, it shows up as a pole of the controller.

17. Recall: Poles on Negative Real Axis



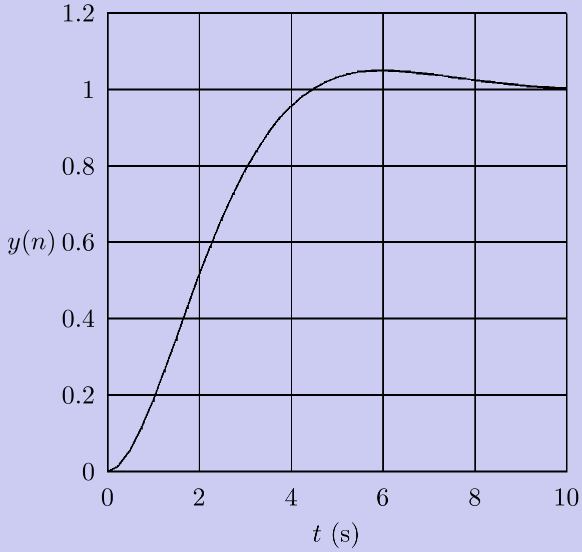
Poles on negative real axis produce oscillations!

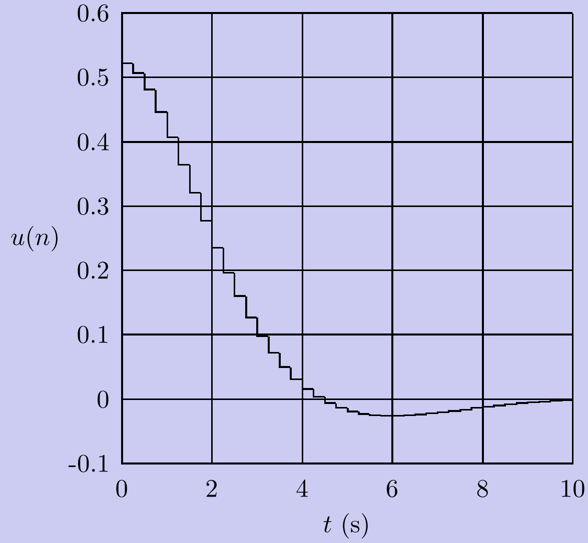
18. Redefining Good and Bad Polynomials

- If B with negative real part is defined as good, it shows up as controller pole
- Whatever is cancelled, shows up in the controller expression
- Preferably, bad factors should lie outside shaded region
- For ease of calculation, negative roots are taken as bad - so don't get cancelled
- A^b, A^g, same as before

$$B^b = 1 + 0.9201z^{-1}$$
$$B^g = 0.0288$$
$$\phi_{cl} = 1 - 1.7502z^{-1} + 0.7791z^{-2}$$
$$R_1 = 1 - 0.7652z^{-1}, \quad S_1 = 0.015$$

19. Redefining Good and Bad Polynomials





Rise time can be improved by shortening the specified rise time from 3s to 2s