CL 692 - Digital Control

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1. Topics to be covered

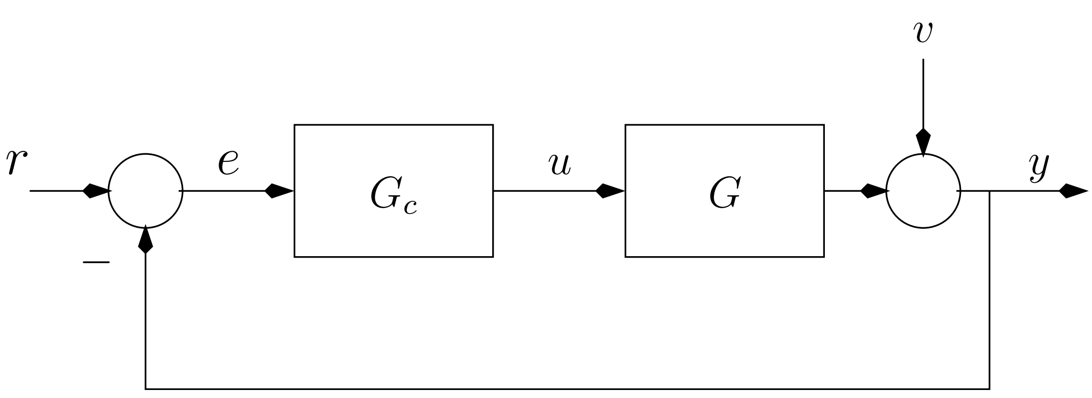
- Modelling
- Signal Processing
- Identification
- Transfer function approach to control design
- State space approach to control design

You will think digital at the end of the course

2. Objectives of a Control Scheme

- To stabilize unstable plants
- To improve the performance of plants (rise time, overshoot, settling time)
- To remove the effect of disturbance (load) and noise
-
- If a good model is available, use feed forward control scheme
- If a good model is not available, use feedback control scheme
- Often, we don't have good models ⇒ feedback control schemes are preferred

3. Schematic of Feedback Control



- G: denotes plant or process
- G_c: denotes digital controller
- r: reference variable, setpoint
- v: disturbance variable
- y: plant output or controlled variable
- u: plant input or manipulated variable or control effort

4. Digital Signals

- Digital systems deal with digital signals
- Digital signals are
  - quantized in value
  - discrete in time
- As 0 or 1 refers to a range of voltages, digital signals can be made less noisy
- Can implement error checking protocols
- So digital devices became popular - impetus for advancement of digital systems
- Digital devices have become rugged, compact, flexible and inexpensive
- Modern controllers are based on digital systems

5. Noise Margin - Impetus for Growth of Digital Devices

source sink

- If transmitted signal is received exactly, no noise
- Analog circuitry always has noise
- Digital devices have good noise margins

Example: TTL

Low Voltage (0)

At output: At input:

Considered low if voltage $<$ 0.4V   Considered low if voltage $<$ 0.8V

- Can pick up noise, voltage can increase by 0.4V, still considered low signal
- Note: if voltage decreases because of noise, no probem

$$\Delta 0 = 0.8 - 0.4 = 0.4V$$

High:

At output At input

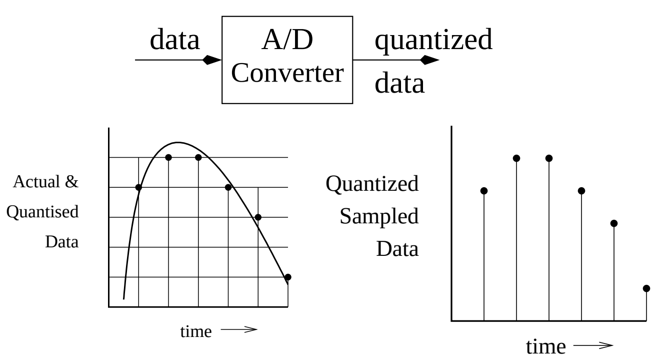
Considered high if voltage $>$ 2.4V   Considered high if voltage $>$ 2V

$$\Delta 1 = 2.4 - 2.0 = 0.4V$$

6. Advantages of Digital Control

- All modern controllers are digital - even if they appear to be analog
- Can easily implement complicated algorithms
- Flexible, can easily change algorithms
- Low prices - can achieve complicated algorithms without much expenditure
- Analysis of difference equations is easier than differential equations - so easier to design digital controllers
- How do we connect digital controllers with real life objects, which could be analog?

7. Analog to Digital Conversion

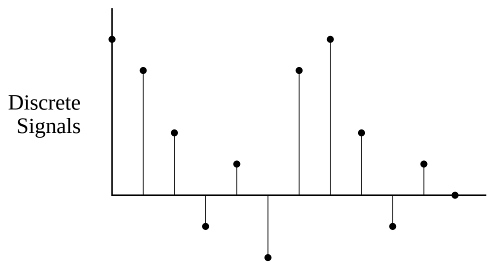


- Analog to Digital (A/D) converter produces digital signals from analog signals
- Higher signal frequency requires faster sampling rate
- But uniform sampling rate is used in an A/D converter

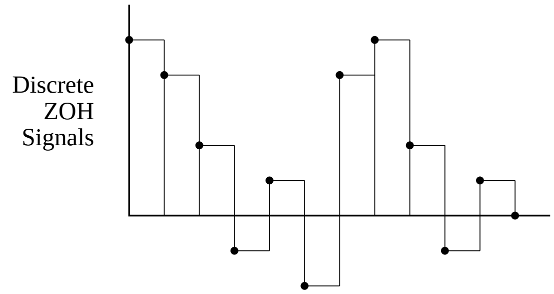
- Quantization errors
  - Finiteness of bits - quantization errors
  - Increase number of bits to reduce errors
  - Falling hardware prices help achieve this
- Sampling rate
  - Slow rate $\Rightarrow$ loss of information
  - Fast rate $\Rightarrow$ computational load
- Analog's output is sent to digital through A/D. Reverse?

8. Digital to Analog Conversion

- Sampled signal

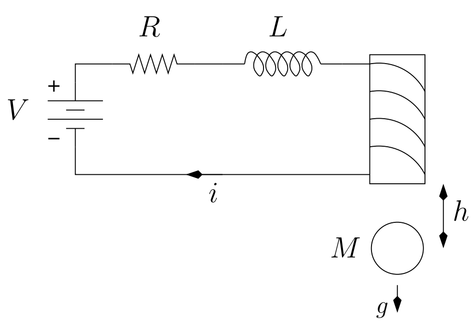


- Real life systems are analog
- Cannot work with binary numbers
- Binary ⇒ D/A ⇒ analog
- Need to know values at all times
- The easiest way to handle this to use Zero Order Hold (ZOH)
- Complicated hold devices possible.



- ZOH is the most popular
- We will consider only ZOH in this course
- Assumption used in this course:
  - All inputs are ZOH signals
  - OK when the input is produced by a digital device
  - Also OK when the input signal varies slowly

9. Magnetically Suspended Ball



- Current through coil induces magnetic force
- Magnetic force balances gravity
- Ball is suspended in midair - 1 cm from core
- Want to move to another equilibrium

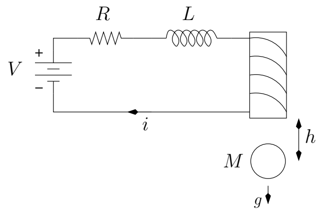
Force balance:

$$M\frac{d^2h}{dt^2} = Mg - \frac{Ki^2}{h}$$

Voltage balance

$$V = L\frac{di}{dt} + iR$$

10. Magnetically Suspended Ball



Model equations:

$$M\frac{d^2h}{dt^2} = Mg - \frac{Ki^2}{h}$$
$$V = L\frac{di}{dt} + iR$$

In deviation variables:

$$0 = M\frac{d^2h_s}{dt^2} = Mg - \frac{Ki_s^2}{h_s}$$
$$M\frac{d^2\Delta h}{dt^2} = -K\left[\frac{i^2}{h} - \frac{i_s^2}{h_s}\right]$$

Linearize RHS:

$$\frac{i^2}{h} = \frac{i_s^2}{h_s} + 2\left.\frac{i}{h}\right|_{(i_s,h_s)}\Delta i - \left.\frac{i^2}{h^2}\right|_{(i_s,h_s)}\Delta h$$
$$= \frac{i_s^2}{h_s} + 2\frac{i_s}{h_s}\Delta i - \frac{i_s^2}{h_s^2}\Delta h$$

Substitute and simplify

$$M\frac{d^2\Delta h}{dt^2} = -K\left[\frac{i_s^2}{h_s} + 2\frac{i_s}{h_s}\Delta i - \frac{i_s^2}{h_s^2}\Delta h - \frac{i_s^2}{h_s}\right]$$
$$\frac{d^2\Delta h}{dt^2} = \frac{K}{M}\frac{i_s^2}{h_s^2}\Delta h - 2\frac{K}{M}\frac{i_s}{h_s}\Delta i.$$

Voltage balance in deviation:

$$\Delta V = L\frac{d\Delta i}{dt} + R\Delta i$$

11. Magnetically Suspended Ball - Continued 3

Force balance:

$$\frac{d^2\Delta h}{dt^2} = \frac{K}{M}\frac{i_s^2}{h_s^2}\Delta h - 2\frac{K}{M}\frac{i_s}{h_s}\Delta i$$

Voltage balance:

$$\Delta V = L\frac{d\Delta i}{dt} + R\Delta i$$

Define new variables

$$x_1 \triangleq \Delta h$$
$$x_2 \triangleq \Delta \dot{h}$$
$$x_3 \triangleq \Delta i$$
$$u \triangleq \Delta V$$

$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = \frac{K}{M}\frac{i_s^2}{h_s^2}x_1 - 2\frac{K}{M}\frac{i_s}{h_s}x_3$$
$$\frac{dx_3}{dt} = -\frac{R}{L}x_3 + \frac{1}{L}u$$

In matrix form:

$$\frac{d}{dt}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ \frac{K}{M}\frac{i_s^2}{h_s^2} & 0 & -2\frac{K}{M}\frac{i_s}{h_s}\\ 0 & 0 & -\frac{R}{L}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}0\\0\\\frac{1}{L}\end{bmatrix}u$$

This is of the form

$$\dot{x}(t) = Fx(t) + Gu(t)$$

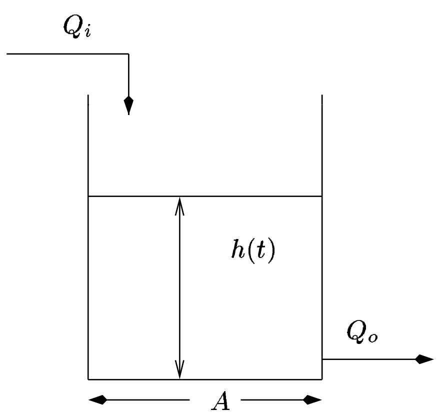
12. Magnetically Suspended Ball - Continued 4

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{K}{M}\frac{i_s^2}{h_s^2} & 0 & -2\frac{K}{M}\frac{i_s}{h_s} \\ 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u$$

M	Mass of ball	0.05 Kg
L	Inductance	0.01 H
R	Resistance	1 Ω
K	Coefficient	0.0001
g	Acceleration due to gravity	9.81 m/s^2
h_s	Equilibrium Distance	0.01 m
i_s	Current at equilibrium	7A

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 981 & 0 & -2.801 \\ 0 & 0 & -100 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} u$$

13. Model of a Flow System



$$A\frac{dh(t)}{dt} = Q_i(t) - k\sqrt{h(t)}$$

Initially at steady state

$$0 = A\frac{dh_s}{dt} = Q_{is} - k\sqrt{h_s}$$

Linearise with Taylor's series approximation:

$$\frac{d\Delta h(t)}{dt} = -\frac{k}{2A\sqrt{h_s}}\Delta h(t) + \frac{1}{A}\Delta Q_i(t)$$

Initial condition:

- at $t = 0$, $h(t) = h_s$ or $\Delta h(t) = 0$.

It can be written as,

$$\dot{x} = Fx + Gu$$

Known as *state space* equation.

14. IBM Lotus Domino Email Server

- Clients access the database of emails maintained by the server through Remote Procedure Calls (RPCs).
- Number of RPCs, denoted as RIS has to be controlled.
- If the number of RIS becomes large, the server could be overloaded, with a consequent degradation of performance.
- If RIS is less, the server is not being used optimally.
- Not possible to regulate RIS directly.
- Regulation of RIS may be achieved by limiting the maximum number of users (MaxUsers) who can simultaneously use the system.
- Because of stochastic nature, difficult to come up with analytic model.
- Obtained through expt., data collection, curve fitting (identification).

$$
\begin{aligned}
y(k) &= \mathrm{RIS}(k) - \overline{\mathrm{RIS}} \\
u(k) &= \mathrm{MaxUsers}(k) - \overline{\mathrm{MaxUsers}} \\
y(k+1) &= 0.43y(k) + 0.47u(k)
\end{aligned}
$$

15. Motivation for Discrete Model

- Real systems are continuous
- Digital controller's view of the process:
  - Receives sampled signals. Sends out sampled signals
  - Thus views the process as a sampled system
- Hence, to determine the control effort, a discrete model of the plant is required
  - Discrete model relates the system variables as a function of their values at previous time instants
  - No value required/used in between sampling instants. Time derivatives have no meaning
- We will next explain how to arrive at discrete models